Working Paper 01-47
Economics Series 10
October 2001

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CYCLICAL BEHAVIOUR OF CONSUMPTION OF NON-DURABLE GOODS: SPAIN VERSUS U.S.A. *

Francisco Xavier Lores †

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JEL Classification: B22, D91, E21, E32.

Keywords: Volatility of Consumption, Domestic Consumption, Imported Consumption.

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***** I wish to thank Luís Corchón, José María Da Rocha, Fernando Del Río, Javier Díaz-Giménez, Juan José Dolado, Jorge Durán, Eduardo Giménez, Carlos Urrutia and, particularly, Michele Boldrin for helpful comments and suggestions. I also benefited from discussions during presentation of earlier versions of the paper at the IV Workshop on Dynamic Macroeconomic at Universidade de Vigo and the Macroeconomics Workshop at Universidad Carlos III de Madrid. I also acknowledge the financial support of Secretaria Xeral de Investigación e Desenvolvemento da Xunta de Galiza by means the program PGIDT00PXI30002PN, the Dirección General de Enseñanza Superior e Investigación Científica y Técnica by means the program SEC99-1094. All errors are, of course, my own.

Cyclical Behaviour of Consumption of Non-Durable Goods: Spain versus U.S.A.*

Francisco Xavier Lores†
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This paper studies the excessive volatility of Spanish aggregate non-durables consumption with relation to aggregate income. In this paper we stress the importance of non-durables imports over the variability of total non-durables consumption and we propose a theoretical model that formalizes the relation between domestic and imported consumption. The main finding of the paper is that a model of intertemporal optimization where there is no credit constrain or government can produce so high volatilities of consumption as the ones observed.

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1 Introduction

There is abundant literature devoted to understand the importance of the decisions about consumption. In the decisions about how much to consume we have to decide how much income we dedicate to consume today versus how much income we keep in order to finance future consumption. The suitable framework for the analysis of these decisions about consumption is the theory of intertemporal choice that formalizes this *trade-off* between present and future consumption explicitly.

Economists agree in the opinion that a basic fact of the behaviour of consumption is that, throughout a period of time, the total amount spent by individuals varies much less than the total income they get. In other words, the aggregated consumption is much less volatile than the aggregated income. Moreover, this fact has led a good part of the theoretical study on consumption.

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One of the most traditional explanations is the so-call *permanent income hypothesis*. The permanent income hypothesis establishes that changes on consumption are basically due to non-expected changes in the permanent income (or long-term income), whereas changes perceived as temporal ones, have a minimum effect. The traditional assumption is that most surprises in the income are temporal and, therefore, consumption must be smoother than the income observed. On the contrary, when the income is modelled with a unitary root, so that temporary changes in the income have permanent effects, these theories may imply a higher volatility of consumption, as Deaton (1987) stated.

A characteristic element of these theories is the lack of link between the return of physical investments and the factors that cause the variations of the aggregated income. Thus, these theories consider the interest rate of investing in capital constant or exogenous. Deaton (1992) is an excellent compendium of these kind of theories.

The link between capital investment income and the factors that cause the movement of the aggregated income is one of the characteristics of the R.B.C. models. In these models the modifications of the aggregated output are governed by innovations that affect the output and, at the same time, the interest rate of the capital investment. Thus the income effect produced by an innovation in the output is partially compensated by the increase in the rate of return of saving. Therefore the consumption response to an unexpected increase of the income is partially compensated by the increase of households saving in order to take advantage of the higher investment return. Both kinds of models, R.B.C. and permanent income belong to a more general kind of models referred to as *equilibrium growth models*[1].

Some examples of R.B.C. models for the Spanish economy are Licandro and Puch (1997), Martín-Moreno (1998), Giménez and Martín-Moreno (2000) and Da Rocha and Restucia (2000). The excessive volatility of the Spanish aggregated consumption with relation to the aggregated income is reported in these articles as well in Dolado et al. (1993) and Lores (2001).

It is necessary to make a first consideration about how the Spanish National Accounting (N.A.) measures the aggregated consumption. The way the aggregated consumption is measured by N.A. differs from what the economic theory considers consumption because it does not include the flow of services generated by durable goods. However, it does include the purchases of durable goods that have a very volatile behaviour which indicate that such purchases are concentrated in the good times. This is the reason why it is convenient to break down the aggregated consumption into its components: consumption of non-durable goods and consumption of durable goods.

Table 1 shows the volatilities relative to the output of the aggregated consumption and its components.

By only observing the consumption of non-durable goods, the excessive volatility with relation to the aggregated income is confirmed. The relative volatility to the output of the consumption of non-durable goods is 0.94 for the 1981-1997 period. Moreover we can observe an increase in such relative volatility in the last 15 years with relation to the 70's and 80's. In fact, both the relative volatility of the aggregated consumption and its components increase.

The prevailing vision in the literature formally mentioned is that these facts suggest a lack of instruments to smoothen consumption intertemporally or that it is the result of frequent changes in taxes and transfers. However, from this point of view the increase of the volatility in the 1986-1997 period is surprising because in this period there is a strong

[1]Sargent (1986) and Hansen (1985) show how the permanent income model is a particular case of the equilibrium growth model.

Serie	1981-1997	1970-1985	1986-1997
Aggregated consumption	1.16	0.93	1.07
Durable consumption	3.85	2.86	3.35
Non durable consumption	0.94	0.70	0.89

Table 1: Relative volatility of consumption. Quarterly data filtered with Hodrick-Prescott. Source: INE-Estrada and Sebastián (1993).

process of liberalization and transformation of the Spanish financing system and there is an opening to the European financial system. Both facts suggest better opportunities to smoothen consumption intertemporally in the last 12 years than in the 1970-85 period. On the contrary, in (Díaz-Giménez, 1999, pag. 252) there is a reference to the possibility that this high volatility of the aggregated consumption is related to the foreign sector of the Spanish economy, that is, if an important part of the aggregated consumption has been produced in the foreign sector, an important part of the fluctuations that affect consumption do not affect the domestic output.

On the other hand, this high volatility of consumption does not seem to be an exclusive feature of the Spanish economy. Other European countries show the same feature. In table 2 the relative volatilities of consumption and its components for different countries of the O.E.C.D. can be compared.

	USA	France	Italy	UK	Spain	Holland
Aggregated consumption	0.7748	0.7500	1.0822	1.1074	1.1609	1.0932
Durable-good consumption	2.9742	4.4399	3.8314	3.5508	3.8581	
Non-durable-good consumption	0.5414	0.5126	0.8058	0.9072	0.9498	
Average of X+M/GDP	0.1896	0.5031	0.3619	0.5015	0.4551	

Table 2: Relative volatility of consumption and degree of openness in same countries. Quarterly data filtered with H-P.1981.1-1997.2. Source: O.E.C.D., I.N.E.

One of the characteristics of the Spanish economy, that became more clear after 1986 when Spain became a member of the E.E.C. is that it is an open economy.

Table 2 also shows a measure of the degree of openness of the economies in some countries of the O.E.C.D. These data simply suggest that there can be some relationship between the degree of openness of the economy and the high volatility of consumption. The European countries, except for France, show a much higher degree of openness as well as higher volatilities of consumptions than the U.S.A.

The statistics presented in tables 1 and 2 were obtained by filtering the series with the H-P filter. Marcet and Ravn (2000) discussed two criteria to modify the H-P filter since they show how the usual choice of the smoothing parameter for quarterly data ($\lambda = 1600$) distorts the results. Although such work focuses on international comparisons of output fluctuations, their conclusions question the aptitude of using the $\lambda = 1600$ value as a smoothing parameter with Spanish quarterly data. These authors propose the following criteria to choose the smoothing parameter: (a) to impose a constraint over the variance of the second difference of the trend with relation to the variance of the cycle, so that such ratio coincides with the one obtained for the U. S. A., and (b) to impose a constraint so that the variance of the second difference of the trend coincides with the one obtained for the U.S.A. These authors state that if the calculation of the cyclical component of the output is adjusted by following these criteria, the volatility of the Spanish output is considerably increased. This outcome suggests the possibility that, by using the same method to obtain the cyclical component of consumption, this cyclical component may be considerably less volatile than the output. The following table presents the relative volatility of the aggregated consumption and the consumption of non-durable goods obtained with the value of $\lambda = 1600$ and the ones proposed by Marcet and Ravn (2000).

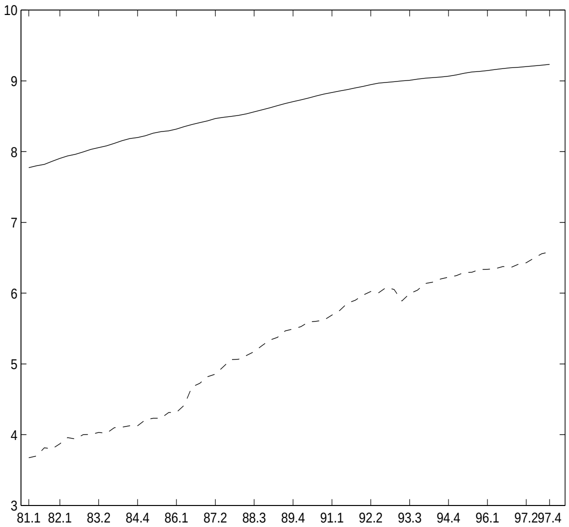
	$\lambda = 1600$	$\lambda = 5385$ case (a)	$\lambda = 6369$ case (b)
Total consumption 70.1-97.4	1.0275	1.1098	1.1215
Non durable consumption 70.1-97.4	0.8555	0.9312	0.9445
Total consumption 70.1-80.4	0.8728	0.9262	0.9417
Non durable consumption 70.1-80.4	0.7603	0.8119	0.8271
Total consumption 81.1-97.4	1.1305	1.2107	1.2219
Non durable consumption 81.1-97.4	0.9201	0.9941	1.0070

Table 3: Relative volatilities of consumption by using the λ proposed by Marcet and Ravn (2000).

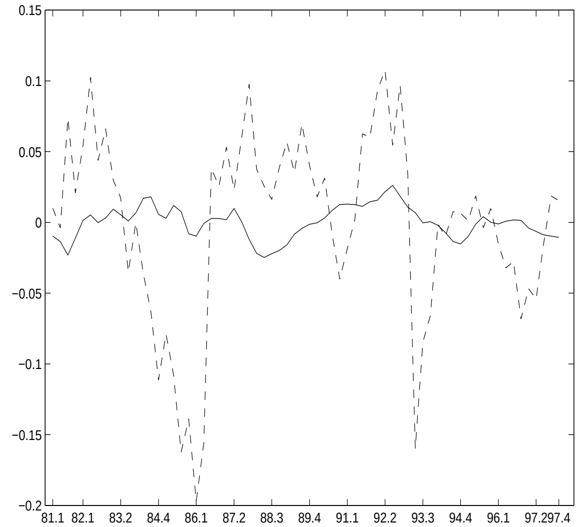
Irrespective of the fact that the criteria proposed by these authors can be debatable, table 1 shows how for the different subsamples, the relative volatility of consumption is higher with the values of the parameters proposed by Marcet and Ravn (2000). The results presented in such table indicate that with the values proposed for λ by these authors, not only the volatility of the output but also of the consumption is increased. Besides that, the increase in the volatility of consumption is higher than that of the income. On the other hand, in Lores (2001) the results of volatility of consumption are presented by using the growth rates and the filter proposed in Baxter and King (1995), as filters, and we obtain similar results to the ones obtained with the H-P filter. Out of this discussion we may conclude that the excessive volatility of consumption is not an artifice caused by the value of the λ parameter in the H-P filter. Given that the commonly adopted value of λ is 1600, in this work we follow this so that the results can be compared.

In the rest of the paper we study the relationship between the degree of openness and the behaviour of the volatility of consumption of non-durable goods in the Spanish economy. Particularly, we will study the effect that imported non-durable consumer goods and their price can have on the behaviour of the total consumption of non-durable goods.

The paper proceeds as follows. Section 2 presents the evidence about the behaviour of domestic and imported non-durable consumption. Section 3 describes the model. Section 4 calibrates the parameters of the model. Section 5 reports the performance of the model via moment comparisons with the Spanish and U.S.A data. Section 6 summarizes the main findings.



1.1: Logarithm of the nominal value of the domestic (−) and imported (−−) non-durable goods.



1.2: Cyclical component of the nominal value of the consumption of domestic (−) and imported (−−) non-durable goods. λ=1600.

Figure 1:

2 Quantifying the Role of the Imported Consumer Goods

The first matter to be solved is how to measure both the amounts and the price of the imported and domestic non-durable consumer goods.

The “consumption of imported non-durable goods at current prices” is measured with “the imports of foods and other non-durable consumer goods series” of the Dirección General de Aduanas del Ministerio de Economía y Hacienda (Customs of the Ministry of Finance and Economic Affairs). Since there is not one series of data of consumption of non-durable domestic goods we have to use the difference between “the current total spending in non-durable consumer goods”, corresponding to the disaggregation of consumption of C.N.E. carried out Estrada and Sebastián (1993), and the “imports of foods and other non-durable goods” of the Dirección General de Aduanas del Ministerio de Economía y Hacienda.

If we separate the “value of the imported non-durable consumer goods” from the “total value of the consumption of non-durable goods” we obtain a different vision of the volatility of consumption in the Spanish economy. Figure 1 shows the two series formerly mentioned and their cyclical component obtained with the H-P filter with $\lambda = 1600$. The different variability that the two series present is highlighted in this figure, which imported consumption is quite more volatile than the domestic one. In order to confirm so, the standard deviations of the cyclical components of the values of domestic and imported consumption are presented in table 2 The latter one has a standard deviation that is six times higher than the domestic consumption.

Obviously, given that the values of the series and not the quantities of the series are being inspected, this different behaviour may be due to the prices. It is necessary to measure these

Cyclical Component	s.d.
Domestic non-durables consumption	0.0112
Imported non-durables consumption	0.0667

Table 4: Standard deviation of the cyclical component of the nominal value of the consumption of domestic and imported non-durable goods.

series in real terms, but the choice of the appropriate prices is not obvious.

2.1 The Price of Domestic Non-Durable Consumer Goods

Figure 2 shows five price indexes that may be a measure of the prices of the domestic non-durable consumer goods. None of them is a "correct" measure of such prices: the "GDP deflator" includes the final prices of all the consumer and investment goods produced in the economy, the "total consumption deflator" contains the prices of the domestic and imported durable and non-durable consumer goods, the "deflator of the consumption of non-durable goods" includes the prices of the imported non-durable consumer goods, the "consumer price index" contains the prices of 471 domestic and imported durable and non-durable consumer goods, whereas the "industrial price index of non-durable consumer goods" shows the prices of the domestic non-durable consumer goods, not including the deliver markup.

Table 5 shows how the cyclical component of these set of prices does not fluctuate much with relation to GDP. The volatility of GDP in the period was 1.13. On the other hand, the cyclical components of these five prices behave in a very similar way as for their volatility and comovement, except for the "industrial price index of non-durable consumer goods" that is more volatile and is the least correlated with the rest of them. That is why we may conclude that any of the price indexes can be a reasonable measure of the price of the domestic non-durable consumer goods, except for the "industrial price index of non-durable consumer goods".

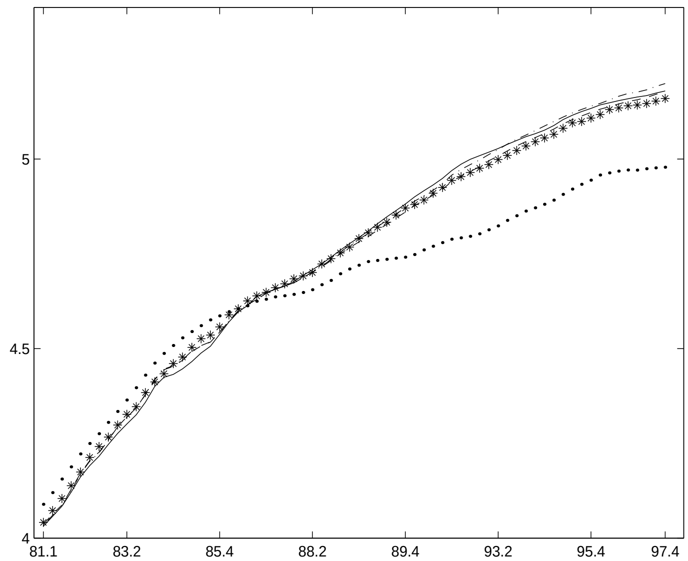


Figure 2: GDP Deflator (−), Private Consumption Deflator (−−), -), Non-Durable Goods Consumption Deflator (−·), CPI (∗), (*), Industrial Price Index of Non-Durable Consumer Goods (·). Source: INE and Estrada and Sebastián (1993).

	DefGDP	DefC	DefCND	CPI	IPINDC
1981.1-1997.4. Standard Deviation (%)					
	1.11	1.18	1.23	1.09	1.55
1981.1-1997.4. Correlations					
DefGDP	1.00 (−)	0.84 (12.76)	0.80 (10.93)	0.76 (9.59)	0.33 (2.89)
DefC	0.84 (12.76)	1.00 (−)	0.94 (22.14)	0.82 (11.53)	0.51 (4.77)
DefCND	0.80 (10.93)	0.94 (22.14)	1.00 (−)	0.79 (10.33)	0.43 (3.91)
CPI	0.76 (9.59)	0.82 (11.53)	0.79 (10.33)	1.00 (−)	0.61 (6.24)
IPINDC	0.33 (2.89)	0.51 (4.77)	0.43 (3.91)	0.61 (6.24)	1.00 (−)

Table 5: DefGDP= GDP Deflator, DefC=Private Consumption Deflator, DefCND=Non-durable Good Consumption Deflator, CPI=Consumer Price Index, IPINDC=Industrial Price Index of Non-Durable Consumer Goods. In brackets: the statistic value $t_{n-2} = \rho(n-2)^{0.5}/(1-\rho^2)^{0.5}$ that is distributed as a t-Student with $n-2$ degrees of freedom.

	NEER	CI	AI	CNDI
1991.1-1998.2. Standard Deviation (%)				
	3.39	2.43	2.57	3.14
Correlations				
NEER	1.00 (−)	0.22 (1.22)	0.34 (1.93)	0.56 (3.56)
CI	0.22 (1.22)	1.00 (−)	0.05 (0.26)	0.55 (3.51)
AI	0.34 (1.93)	0.05 (0.26)	1.00 (−)	0.50 (3.08)
CNDI	0.56 (3.56)	0.55 (3.51)	0.50 (3.08)	1.00 (−)

Table 6: NEER= Nominal effective exchange rate, CI=Unitary value index of the imports of consumer goods, AI=Unitary value index of the imports of foods, CNDI=Unitary value index of the imports of non-durable consumer goods. In brackets $t_{n-2} = \rho(n-2)^{0.5}/(1-\rho^2)^{0.5}$.

2.2 The Price of Imported Non-Durable Consumer Goods

In order to measure the imported non-durable consumer goods there is a price index that is the "unitary value index of the imports of non-durable consumer goods", built by the Dirección General de Aduanas del Ministerio de Economía y Hacienda. Unfortunately it is only available for a very small sample, (1991.1-1998.2). There are several alternative price indexes with appropriate sample sizes. They are the following: the "nominal effective exchange rate", the "unitary value index of the imports of consumer goods" and the "unitary value index of imports of foods". Figure 3 shows the alternative price indexes, whereas in figure 4 the same price indexes are represented with the unitary value index of the imports of non-durable consumer goods for the 1991.1-1998.2 sample.

Unlike what happened with the price of domestic non-durable consumer goods, in the case of the price of imported non-durable consumer goods we do have a "correct" index, but only for the 1991.1-1998.2. It could be reasonable to choose the index of the alternative prices that has the most similar behaviour to the "correct" one in this seven-year period as a measure of the prices of imported non-durable consumer goods.

Table 6 presents the standard deviations and the correlations among the cyclical components of all these price indexes. All of them present a similar volatility and correlations of around 0.5 with the price index of imported non-durable goods.

Given that the behaviour of the different price indexes is very similar, both for domestic and imported goods, the final choice of the pair of price indexes to deflate the series is to use the non-durable good consumption deflator for the domestic consumption series and the unitary value index of the imports of foods for the imported consumption series.

In the following section we use this price indexes and the current values of the series of spending in consumption of non-durable goods to describe the behaviour of the quantities and the relative price.

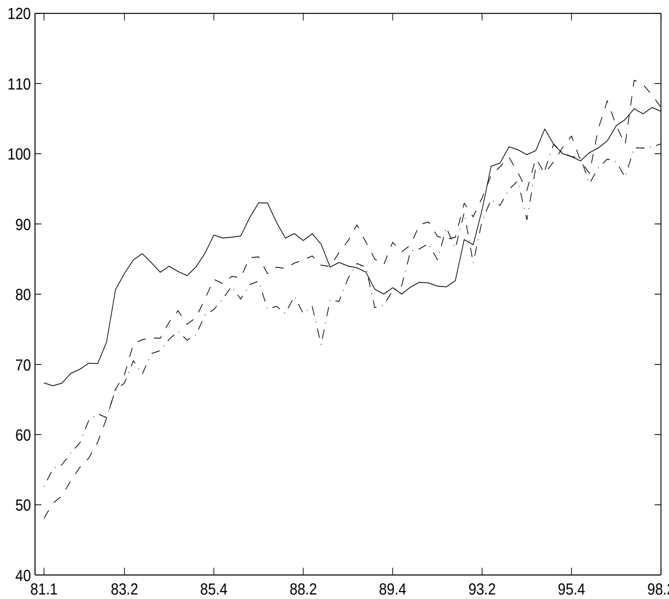


Figure 3: Nominal effective exchange rate with the countries in the O.E.C.D. (−), unitary value index of the imports of consumer goods (−−), unitary value index of the imports of foods (−·).

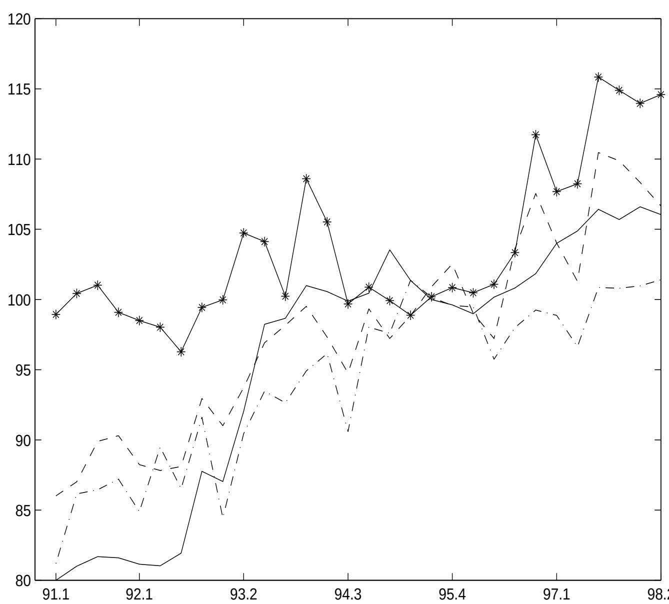


Figure 4: Nominal effective exchange rate with the countries in the O.E.C.D. (−), unitary value index of the imports of consumer goods (−−), unitary value index of the imports of foods (−·), unitary value index of the imports of non-durables consumer goods (∗).

Series	s.d (%)	σ_x/σ_{gdp}	corr(pib,x)
GDP	1.129	1	1
Non-durable good total consumption	1.038	0.91	0.79
Non-durable good domestic consumption	1.016	0.89	0.62
Non-durable good imported consumption	7.665	6.78	0.48
Imported goods/domestic goods ratio	7.262	6.43	0.43
Relative price (AI/DefCND)	2.599	2.30	-0.11

Table 7: Spanish Economy. Statistics of the cyclical components of the amounts and relative price. Domestic price=Deflator of the consumption of non-durable goods and foreign price=unitary value index of the imports of foods. H-P filtered. 1981.1-1997.4.

2.3 The Behaviour of Quantities and the Relative Price

The series of consumption of imported and domestic non-durable goods are obtained by deflating with the unitary value index of the imports of foods and the deflator of consumption of non-durable goods respectively. The ratio between these indexes will be used as a measure of the relative price of the imported and domestic non-durable consumer goods. Thus, table 7 shows statistics of the deviations of the trend of these series that will be used as a set of reference data.

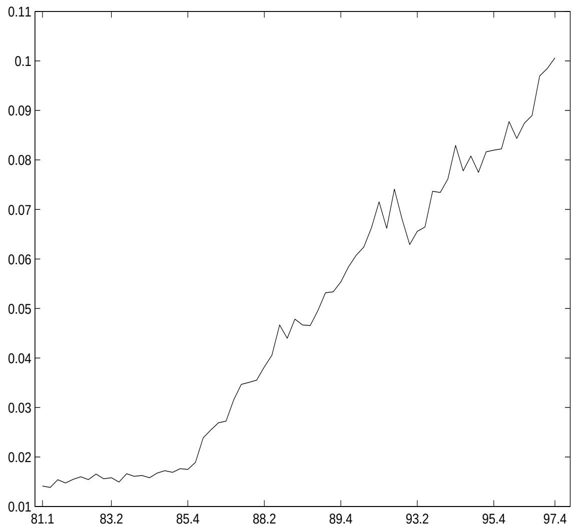
What was observed in the series of values is confirmed with the series in real terms. The imports of non-durable consumer goods are much more volatile than the consumption of domestic non-durable goods, that is almost seven times as volatile as the former ones. The relative price of these goods is also very volatile, more than twice as volatile as GDP. On the other hand, the relative price behaves countercyclically, that is, in good times foreign non-durable consumer goods tend to become cheaper than domestic ones. At the same time, the ratio between imported and domestic goods is also procyclically and presents a -0.41 coefficient of correlation with the relative price.

Table 8 shows the relative volatilities of these series and their correlations with GDP when other pairs of price indexes are used to deflate the series and built the relative price. The behaviour discussed above is kept for all of them. The relative volatilities of the imports and the imported/domestic goods ratio move in the $[4.14; 7.32]$ and $[4.04; 7.04]$ rank respectively, whereas the volatility of the relative price moves in the $[1.85; 3.47]$ rank, showing a negative correlation with the deviations of the GDP of its trend in all the cases but one.

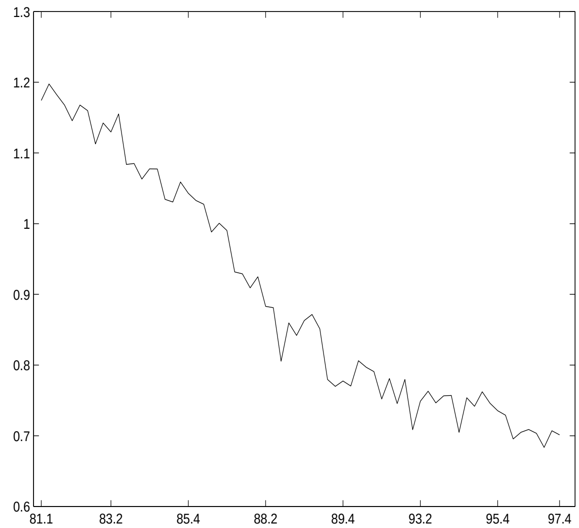
These observations suggest a cyclical behaviour of the consumption of non-durable goods in keeping with what the theory of the consumers choice suggests. Since it is about non-durable consumer goods it is reasonable to think that they are quite substitutive goods. In fact in next sections there is an estimation of the value of the elasticity of substitution among these goods and there is a result that confirms this intuition. Thus, when the imported consumer goods become cheaper than the domestic ones, consumers will substitute part of their domestic consumption for imported one, increasing the share of foreign goods in their consumption bundles. But, if, as the data suggest, the cheapening of the imported goods coincides with periods in which the aggregated income increases and the goods are not perfect substitutive ones, the total consumption of non-durable goods will increase more than if the relative price of the goods have not been changed.

	P_1		P_2		P_3		P_4		P_5		P_6	
	σ_x/σ_{gdp}	$\rho_{gdp,x}$	σ_x/σ_{gdp}	$\rho_{gdp,x}$	σ_x/σ_{gdp}	$\rho_{gdp,x}$	σ_x/σ_{gdp}	$\rho_{gdp,x}$	σ_x/σ_{gdp}	$\rho_{gdp,x}$	σ_x/σ_{gdp}	$\rho_{gdp,x}$
Total consumption	.77	.79	.76	.77	.75	.79	.77	.92	.84	.69	.84	.69
Domestic consumption	.76	.63	.73	.62	.73	.62	.70	.88	.79	.63	.79	.63
Imported consumption	5.84	.44	5.80	.53	5.55	.48	4.14	.27	7.09	.43	7.32	.59
Imported/domestic ratio	5.54	.38	5.48	.48	5.26	.43	4.04	.21	6.92	.38	7.04	.54
Relative price	1.85	-.19	1.96	-.15	1.88	-.11	2.30	.08	2.41	-.17	3.47	-.42

Table 8: Relative volatilities to GDP and correlations of the series using as deflators: P_1=U.V.I. of imports of consumer goods/Def. total consumption, P_2= P2=U.V.I. of imports of foods/C.P.I., P_3= U.V.I. of imports of foods/Def. consumption of non-durable goods, P_4= U.V.I. of imports of non-durable consumer goods/Def. consumption of non-durable goods, P_5= U.V.I. of imports of consumer goods/GDP Def, P_6= NEER/GDP Def.



5.1: Ratio between consumption of imported and domestic non-durable goods.



5.2: Relative price (AI/Def CND).

Figure 5: Spanish Economy.

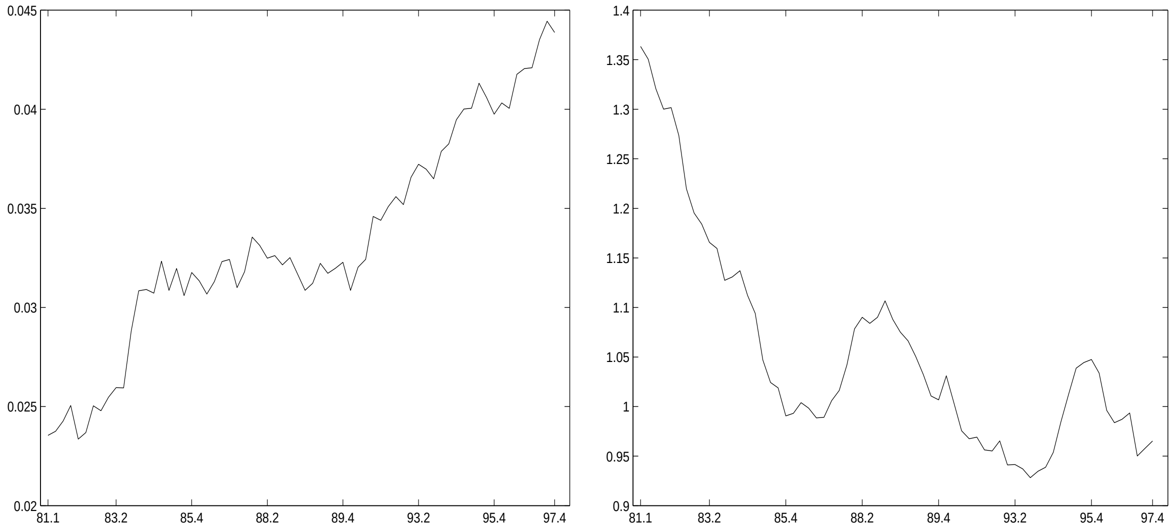
Figure 5 allows us to confirm that the evolution in time of the level of the relative price and the ratio of imported non-durable consumer goods over the domestic ones is also coherent with the theory. From the early 80s the relative price of these goods has been experiencing a continuous fall, whereas the ratio of imported over domestic goods experiences a continuous growth, especially since 1986, when there is a strong openness of the Spanish economy.

In order to compare with the Spanish economy we can see what happens in the U.S. economy in the same period of time and the same aggregated variables.

Table 9 shows the same statistics shown for Spain in table 7. The data used for the U.S.A were built in the same way as for Spain, by using the implicit deflator of the consumption of non-durable goods and a price index of the imported non-durable consumer goods as deflators for the domestic and imported consumption. All these data were obtained from the National Income Product Accounts (NIPA) elaborated by the Bureau of Economic Analysis of U.S. Department of Commerce (BEA).

Like in the Spanish economy, the imports of non-durable consumption are more volatile than the product although the relative volatility is not as high as in the Spanish economy. The same thing happens with the imported/domestic goods ratio. However, the relative price of these goods has a higher volatility than the relative price for the Spanish economy in table 7. Moreover, the sign of the correlation with the product is a positive one.

On the other hand, figure 6 shows the series of the imported/domestic good ratio and the relative price for the U.S.A. We can observe a gradual increase in the proportion of imported goods over domestic ones and a fall in the relative price like in the Spanish economy. However, the size that the ratio of the imported goods over the domestic ones reaches at the end of the sample is much smaller in the U.S.A. Taking similar levels in 1991 as a starting point, the proportion of imported goods over domestic goods in the Spanish economy reaches 10%



6.1: Ratio between imported and domestic non-durable consumer goods for the U.S.A.

6.2: Relative price (AI/Def CND) for the U.S.A.

Figure 6: U.S.A. Economy

Series	s.d (%)	σ_x/σ_{pib}	corr(gdp,x)
GDP	1.428	1	1
Non-durable good total consumption	0.9117	0.63	0.76
Non-durable good domestic consumption	0.869	0.60	0.71
Non-durable good imported consumption	3.746	2.62	0.58
Imported goods/domestic goods ratio	3.567	2.49	0.44
Relative price	3.765	2.63	0.17

Table 9: U.S.A. Economy.Statistics of the cyclical components of the amounts and relative price. Domestic price=Deflator of the consumption of non-durable goods and foreign price=Price index of the imports of non-durable consumer goods. H-P filtered 1981.1-1997.4.

in 1997, whereas in the U.S.A it hardly reaches 4.5%.

3 A Theoretical Explanation

The intuitions in the former section are modelled in this section by means of a model that is capable to generate a behaviour of the deviations of the trend of consumption of non-durable goods as the one observed. A model of consumption-saving choice in which the interest rate and the relative prices are considered exogenous by a representative agent is proposed. The model considers two consumer goods: one of them is produced outside the economy and the other one inside. The representative agent exchanges domestic consumer good for foreign consumer good at a rate that is determined in the international markets and, therefore, will be considered exogenous to the model and evolves stochastically. The foreign consumer good is only used to consume, that is, investment goods are not imported.

The preferences of the representative agent have the following shape

$$E\left\{\sum_{t=0}^{\infty}\delta^t\frac{\left[aC_t^{\sigma}+(1-a)\tilde{C}_t^{\sigma}\right]^{\frac{1-\gamma}{\sigma}}}{1-\gamma}\right\}$$

with $\gamma > 0,\ \gamma \neq 1,\ \sigma \leq 1,\ \sigma \neq 0$ where $0 < \delta < 1$ is the discount factor, C_t consumption of domestic goods, $\tilde{C}_t$ consumption of imported goods, σ is the parameter related with the elasticity of substitution between both goods, γ the parameter related with the elasticity of intertemporal substitution of the consumption and $0 < a < 1$.

The evolution of the level of accumulated assets is expressed by

$$K_{t+1} = (A_t + 1 - \mu)K_t - C_t - p_t\tilde{C}_t$$

where A_t evolves by means of an equation in stochastic differences $A_{t+1} = \phi(A_t, \varepsilon_{t+1})$ with ϕ a continuous function and ε_t a random variable *i.i.d.* with support in $\mathcal{R}$ mean 0 and variance σ_ϵ^2. p_t represents the relative price of both goods and evolves by following an equation in stochastic differences $p_{t+1} = \theta(p_t, u_{t+1})$ where θ is a continuous function and u_t a random variable *i.i.d.* with support in $\mathcal{R}$ media 0 and variance σ_u^2. $0 < \mu < 1$ is the rate of capital depreciation.

Thus, the problem of the intertemporal choice of the representative agent is

$$\max_{C_t,\tilde{C}_t,K_t} E\left\{\sum_{t=0}^{\infty}\delta^t\frac{\left[aC_t^{\sigma}+(1-a)\tilde{C}_t^{\sigma}\right]^{\frac{1-\gamma}{\sigma}}}{1-\gamma}\right\} \tag{1}$$

s.a:

$$\begin{aligned}
&C_t + p_t\tilde{C}_t = (A_t + 1 - \mu)K_t - K_{t+1}\\
&A_{t+1} = \phi(A_t, \varepsilon_{t+1})\\
&p_{t+1} = \theta(p_t, u_{t+1})\\
&K_0, A_0, p_0 \ \text{ dados}
\end{aligned}$$

The model tries to focus on the most relevant aspect of the problem, that is, the excessive volatility of the aggregated consumption is a puzzle for the theory of the intertemporal choice of consumption. The main matter to be solved is whether a more volatile sequence of consumption than the income is compatible with the intertemporal optimization of a concave utility function. This approach is also relevant for the theory of real business cycles because, although the suggested model is not a model of the Spanish economic cycle, standard R.B.C theories are also based on the theory of intertemporal choice of consumption. Despite a crucial aspect of economic cycles are fluctuations in the labour market, labour supply is not considered endogenous here, but this characteristic will not bias the results in favour of the thesis suggested. Consumption and leisure are usually substitutive goods in the utility function. If we considered a model with exogenous productivity shocks, leisure would be reduced (increased) in periods of high (low) productivity, which would cause increases in consumption but never reductions. On the other hand, Solow residual productivity shocks are not used here as exogenous impulses, but changes in the rate of return of capital and changes in the real exchange rate. The volatility of the income will be governed by the volatility of the rate of return that will be calibrated so that it coincides with the volatility observed of GDP. Likewise, in a model with endogenous labour supply, the calibration must be adjusted to reproduce the volatility of the output, so, the volatility of the output would not be higher than in the model proposed either.

On the other hand, the interpretation of the trade balance in this model will be the positive or negative variation of the net position of assets. The Spanish economy is and was a small open economy in the period considered. That is the reason why the rate of return must be equalized to the rate of the international rate of return. When the representative agent consumes more than he has, he reduces its capital which, in this model implies a deficit with the rest of the world. The task in this chapter is not to explain the reasons why the real exchange rate behaved the way it did, but that the movements of the real exchange rate are used to explain the volatility of consumption and its composition. Actually, we use the movements of an available empiric measure of the relative prices of imported consumer goods versus domestic ones.

3.1 An Interpretation of the Model

On the other hand, from the point of view of the models of equilibrium growth, this model can be interpreted as a model where the evolution of the production is not supposed to present decreasing returns in capital. This model is known in literature as the AK model. The standard justification is to consider the capital of the economy more extensively in order to include not only the physical capital but also the human, public capital or knowledge. McGrattan (1998) uses this model where there is evidence of the positive relation between saving rates and growth rates which is considered as evidence in favour of this kind of models.

The simplest version that makes this interpretation explicit is to consider a technology with two production factors: physical capital and human capital[2]

$$Y = ZF(K, H)$$

where F is a neoclassical production function and Z is a stationary stochastic process with support in $\mathcal{R}^+$. We can rewrite the production function in intensive terms by means of the property of constant returns to scale

$$Y = KZf\left(\frac{H}{K}\right) \tag{2}$$

The production of the only good can be used to consume or invest in any of the two capitals. The same technology is used to produce physical capital or human capital and the rates of capital depreciation are μ_k and μ_h respectively. If R_k and R_h are the rent gross prices of

[2]A detailed discussion on the model can be found in Barro and Sala-i Martin (1995).

the respective capitals paid by competitive firms, in competitive equilibrium the marginal product of each product equalizes these prices

$$Z\left[f\left(\frac{H}{K}\right)-\frac{H}{K}f'\left(\frac{H}{K}\right)\right]=R_k$$
$$Zf'\left(\frac{H}{K}\right)=R_h$$

in equilibrium the rates of return of the owners of the capitals $R_k-\mu_k$ and $R_h-\mu_h$ have to be equal. This equality implies

$$Z\left[f\left(\frac{H}{K}\right)-f'\left(\frac{H}{K}\right)\left(1+\frac{H}{K}\right)\right]=\mu_k+\mu_h$$

given the properties of the neoclassical production function this condition determines H/K as a function of Z

$$\frac{H}{K}=h(Z)$$

By defining $A=Zf(h(Z))$ we conclude that equation (2) implies an AK technology that experiences stationary shocks. This result suggest that, for the AK model to be a good representation of the growth process of an economy, we should observe that the H/K ratio is stationary.

Although there are no available measures of human capital stock we can use expression (2) that relates the H/K ratio with the ratio of the production over physical capital stock, so that if H/K is stationary, Y/K should be so too.

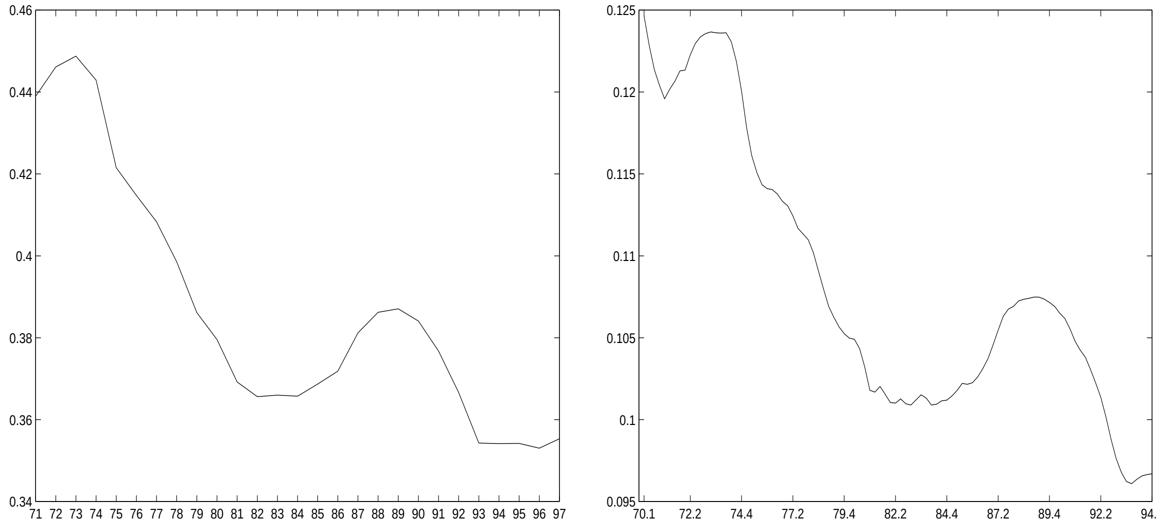
Figure 7 shows the ratio between GDP and physical capital stock of the Spanish economy. The capital stock series of BBV was used to calculate the yearly Y/K ratio. In order to calculate a quarterly capital stock series the yearly data were supposed to correspond with the capital stock of the last quarter of the corresponding year and the rest of the observations are interpolated by using Gómez and Maravall (1997), that basically makes an interpolation by means of optimum forecasts in view of the available observations.

On observing the figures it seems clear to conclude that such a ratio presents a very strong trend in the 70s and a more stable behaviour with cyclical fluctuations since the early 80s. Thus, it may show a steady state change experienced by the Spanish economy in the 70s. Since this work focuses on the posterior period to 1981 the stationarity of the Y/K ratio for the period is assumed.

3.2 Stochastic Trends

Another characteristic of the model proposed is that it implies the existence of stochastic trends in the variables of the economy. Consider, for instance, the solution of this model with a good when A_t is $i.i.d$ that can be found in the appendix. It is easy to check that the optimum paths of the income, consumption, capital and investment follow a random walk with drift

$$\begin{aligned}
&\log(Y_t)-\log(Y_{t-1})=\log(\alpha)+\log(A_{t-1})\\
&\log(C_t)-\log(C_{t-1})=\log(\alpha)+\log(A_{t-1})\\
&\log(K_t)-\log(K_{t-1})=\log(\alpha)+\log(A_{t-1})\\
&\log(I_t)-\log(I_{t-1})=\log(\alpha)+\log(A_{t-1})+\log(\alpha A_t-1+\mu)-\log(\alpha A_{t-1}-1+\mu)
\end{aligned}$$



7.1: GDP/physical capital stock ratio. Yearly series.

7.2: GDP/physical capital stock ratio. Quarterly series

Figure 7: Source: CNE, BBV and own computations.

where α is a constant, $\alpha = \left(\delta E\left[A_t^{1-\gamma}\right]\right)^{1/\gamma}$. Although the fact that there are or there are not stochastic trends in the macroeconomic series is an open matter there is evidence of a unitary root for the Spanish GDP (see, for example Martínez and Espasa (1998)).

The implications of the presence of stochastic trends in macroeconomic series are important. If the trend is a stochastic one and receives a shock, this implies a permanent change in the level of the trend, therefore, of the series. In other words, the trend does not tend to go back to a certain level after a shock.

From the point of view of the economic modelling, the presence of a stochastic trend is related to the theory of stochastic growth. Two kinds of models of stochastic growth, according to the stochastic trend they generate, can be considered. On the one hand, models in which the presence of permanent exogenous productivity shocks is the only responsible of the existence of the stochastic trend. This is the case of the R.B.C. standard models when, for example, the growth of the total factor productivity follows a random walk. King et al. (1991) discuss these kind of models and their implications for some U.S macroeconomics series.

On the other hand, there are models in which the stochastic nature of the growth is the result of the effects that fluctuations have on growth, that is, the stochastic properties of the trend are not exogenous but the result of endogenous responses of the technology to fluctuations. In these models any temporal disturbance causes permanent effects in the level of production insofar as it produces temporary changes in the amount of resources allocates to grow. Fatás (2000) discusses the implications of this kind of models and suggests a test to contrast empirically what kind of models is more appropriate. His results suggest the aptitude of the second kind of models.

The model proposed here belongs to this second kind of models. The model generates unitary roots in the series because shocks have an effect over the capital accumulation. After that cyclical effect disappears the level of the series does not go back to a trend level. This means that shocks have permanent effects. A characteristic of this kind of models is that any kind of shocks causes a similar answer in the series. In the model proposed, the shocks are the capital return due to the similarity with the R.B.C. standard models, but other kind of shocks (for example: shocks of aggregate demand) would cause a unitary root in the series of the model.

On the other hand, the model does not have any other implication over the variables of the labour market, so it is compatible with any dynamic behaviour in the labour market.

4 Assigning Values to the Parameters

In this section we discuss the values assigned to the parameters of the model. The strategy is to obtain values of the parameters by using external information or a subset of moments of the data, to simulate the model and validate it subsequently by means of a different subset of moments from the one used to obtain values of the parameters.

The values of the parameters of the stochastic processes A_t and p_t are chosen to reproduce the observed behaviour of the measures of the relative price of goods and the average productivity of the capital discussed in sections 2.3 and 3.1 respectively. The parameter of the elasticity of substitution between goods σ and the parameter a of the utility function are estimated by using the first order conditions of the model —expression (5) in appendix—. The parameter of the elasticity of intertemporal substitution γ is obtained from literature. The capital depreciation rate μ is chosen so that it coincides with the depreciation rate implicit in the capital stock series of Mas et al. (1996). Finally, the discount factor δ is chosen so that the model reproduces the average growth rate of the capital stock series in the 1981.1-1997.4.

4.1 The Capital Depreciation Rate

The capital depreciation rate is obtained by regressing the data of the capital and investment stock of B.B.V. for the 1964-1994 period

$$K_t^{bbv} - I_t^{bbv} = (1 - \mu^{bbv}) K_{t-1}^{bbv}$$

The resulting depreciation rate was $\mu^{bbv} = 0.0574$ that corresponds to a quarterly depreciation rate $\mu = 0.0147$.

4.2 The Elasticity of Substitution between Goods

Expression (5) relates the ratio between consumption of imported and domestic non durable goods with the relative price of both kinds of goods. Such an expression is easily linealized by taking logarithms, so that we can write

$$\log\left(\frac{\tilde{C}_t}{C_t}\right) = B + D\log(p_t) \tag{3}$$

where

$$B = \frac{1}{\sigma - 1}\log\left(\frac{a}{1-a}\right),\ D = \frac{1}{\sigma - 1}$$

Dickey-Fuller Regression: $\Delta x_t = \mu_0 + \phi x_{t-1} + \rho_1 \Delta x_{t-1} + \ldots + \rho_5 \Delta x_{t-5} + \epsilon_t$		
Variable	ϕ Estimated	t-ratio
$\tilde{C}_t/C_t$	-0.007889	-0.662785
p_t	-0.034856	-1.495625
MacKinnon critical values for the Dickey-Fuller test:		
	-3.5380 al 1%	
	-2.9084 al 5%	
	-2.5915 al 10%	

Table 10: Test of unitary root for the relative price and the proportion of goods. The sample period is 1981.1-1997.4.

Eigenvalue	R-Likelihood	Critical V. 5%	Critical V. 1%	H_0:Num. EC
0.194339	21.85142	19.96	24.60	None
0.142960	9.101957	9.24	12.97	At lest one

Table 11: Cointegration test between relative price and proportion of goods. The sample period is 1986.1-1997.2.

the model imposes a linear relation in the logarithms of the variables previously mentioned. This relation can be used to obtain a value of the parameter of the elasticity of substitution between goods.

On the other hand, in figure 5 we can see how the behaviour of the proportion of goods and relative price has a trend. Particularly, there may be unitary roots in such series. Table 10 shows the results of a test to see if there are unitary roots in such series. The null hypothesis that there is a unitary root in the series can not be rejected for the three significance levels. So, if we want to use the relation (3) to obtain an estimation of the parameter σ, it is necessary to think of the possibility that the series may be cointegrated.

If a series has to be differentiated d times for it to be a stationary one, then it presents d unitary roots and it is said to be integrated of order d, denoting I(d). If we consider two series y_t and x_t both of them I(d) ones, any linear combination of the two series will also be I(d). However, when there is a β vector such that the term of error in the regression of y_t over x_t ($\epsilon_t = y_t - \beta x_t$) is of a lower order of integration than y_t and x_t, I(d-b) with $b > 0$, Engle and Granger (1987) defined y_t and x_t as cointegrated of order (d,b), and is denoted CI(d,b).

The economic interpretation of the cointegration is that if two series are related by means of a relationship of long-term equilibrium, although the series can contain stochastic trends (that is, they are not stationary), they will move together in time and the difference between them will be stable (stationary). Then, the relation of cointegration ($y_t = \beta x_t$) implies the existence of a long-term equilibrium among the variables that converge in time, and ϵ_t is interpreted as an error of disequilibrium in the t moment.

The Johansen test is applied to contrast the cointegration between p_t and $\tilde{C}_t/C_t$. The results of such test are shown in table 11.

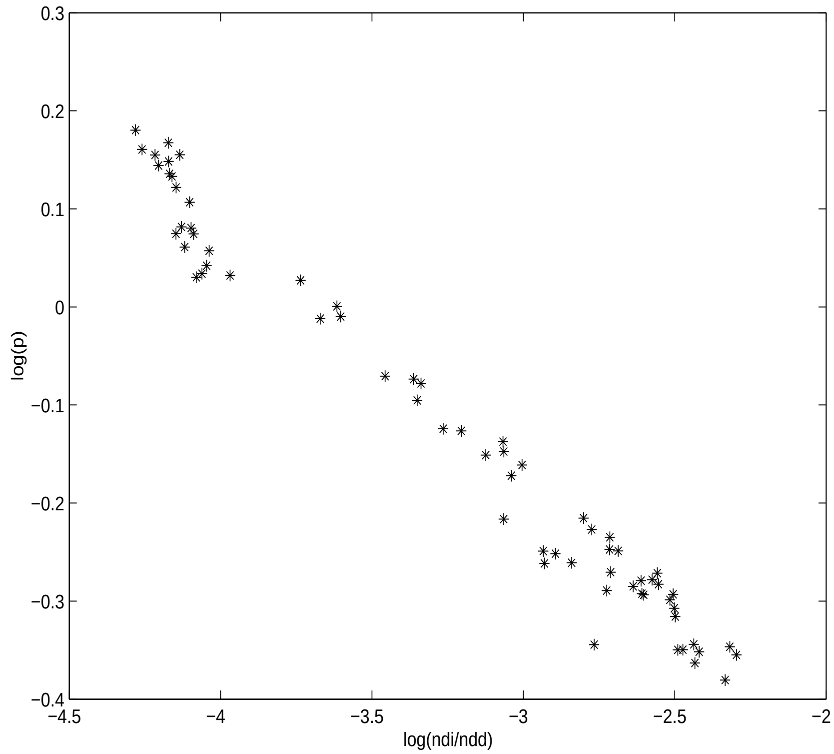


Figure 8: Relation between the logarithms of the relative price and the ratio of imported and domestic non durable consumer goods.

The values of the cointegration test indicate that the existence of cointegration of the variables can not be rejected. Thus, to estimate the relation (3) by OLS when the variables are cointegrated does not show any problem asymptotically. However, in small samples the bias can be a problem. So that the relation (3) is estimated as a relation of cointegration between the variables. The estimated cointegration equation is

$$\log\left(\frac{\tilde{C}_t}{C_t}\right) = \underset{(0.3677)}{-3.0762} \underset{(0.9929)}{-2.1063} \log(p_t)$$

that provides estimates for $\sigma = 0.5252$ and $a = 0.8116$. Figure 8 represents the logarithm of the proportion of the consumptions of goods versus the logarithm of the relative price. In it we can appreciate the strong linearity of the relation between both variables.

4.3 The stochastic processes of A_t and p_t

The processes of A_t and p_t are obtained by estimating (12) by O.L.S. for the 1981.1-1997.4 period. In this case we adjust autoregressive processes for the rate of return and the relative price without taking into account the unitary root that presents the relative price. The reason is that to provide the model with a stochastic process of the relative price or the rate of return with a unitary root implies to admit that the level of the relative price or the rate of return are not bounded. In particular, the existence of a decreasing stochastic trend in the relative price would allow the relative price to take null or negative values which does not make economic sense.

The parameters of the estimated autoregressive processes were[3]

$$\begin{pmatrix} \phi_1 & 0 \\ 0 & \theta_2 \end{pmatrix} = \begin{pmatrix} \underset{(0.01225)}{0.9857} & 0 \\ 0 & \underset{(0.0111)}{0.9749} \end{pmatrix} \quad \begin{pmatrix} \overline{A} \\ \overline{p} \end{pmatrix} = \begin{pmatrix} \underset{(0.0288)}{-0.0343} \\ \underset{(0.0032)}{-0.0103} \end{pmatrix}$$

the matrix of variances and covariances of (ε_t, u_t) is chosen so that the model can reproduce the matrix of variances and covariances observed Γ of $(\log(A_t), \log(p_t))$

$$\Gamma = \begin{pmatrix} \sigma^2_{\log(A)} & \operatorname{cov}(\log(A), \log(p)) \\ & \sigma^2_{\log(p)} \end{pmatrix} = \begin{pmatrix} 0.000965 & -0.000706 \\ & 0.02779 \end{pmatrix}$$

that is,

$$\Sigma = \Gamma - \begin{pmatrix} \phi_1 & \phi_2 \\ \theta_1 & \theta_2 \end{pmatrix} \Gamma \begin{pmatrix} \phi_1 & \phi_2 \\ \theta_1 & \theta_2 \end{pmatrix}^{\top}$$

The stochastic processes of $\log(A_t)$ and $\log(p_t)$ imply a average or long-term value for the variables mentioned given by

$$\begin{pmatrix} \log(A^e) \\ \log(p^e) \end{pmatrix} = \left[I - \begin{pmatrix} \phi_1 & \phi_2 \\ \theta_1 & \theta_2 \end{pmatrix} \right]^{-1} \begin{pmatrix} \overline{A} \\ \overline{p} \end{pmatrix}$$

so that $A^e = 0.0908$ and $p^e = 0.6610$.

In the simulations we take $A_0 = 0.0904$ and $p_0 = 1.1741$ that correspond to the values observed in the first quarter of 1981. These initial conditions are quite far from the stationary values of the stochastic processes, specially that of the relative price. The choice of these initial conditions is motivated by the observation that the behaviour of the relative price and proportion of consumption of goods is not stationary. Figure 5 shows clearly how both relative price and proportion of consumption present clear trends. On the other hand, it does seem reasonable to think that these variables do not tend to stabilize sometime. So with this choice of the initial conditions the model is obliged to start from a situation that is as similar to the Spanish economy in 1981 as possible. On the other hand, the solution method used to solve the model is coherent with this choice.

4.4 Discount Factor

Finally, the discount factor is chosen so that the growth rate of the capital stock in steady state coincides with the average growth rate of the capital stock in the 1981.1-1994.4 period, that is

$$\left[\delta \left(A^e + 1 - \mu\right)\right]^{\frac{1}{\gamma}} = 1.0029$$

The γ parameter associated with the elasticity of intertemporal substitution of the consumption of non durable goods, was taken from Hahm (1997) that estimates a 2.77 value for γ with data of consumption of non durable goods for the U.S.A.

[3] In the initial estimation crossed parameters between the variables that are different from zero were allowed. However, they were not statistically different from zero.

Preferences		
Discount Rate	δ	0.9368
Elasticity of Substitution $(1/\sigma - 1)$	σ	0.5252
Elasticity of Intertemporal Substitution$(1/\gamma)$	γ	2.77
Domestic Consumption Weight	a	0.8116
Depreciation rate	μ	0.0147
Stochastic Processes		
Constant of the process of capital return	$\overline{A}$	-0.0343
Capital return autocorrelation coefficient	ϕ_1	0.9857
Standard deviation of shock of capital return	σ_ε	0.0052
Constant of the process of the relative price	$\overline{p}$	-0.0103
Relative price autocorrelation coefficient	θ_1	0.9749
Standard deviation of shock of relative price	σ_u	0.0374
Covariance between shocks	$\sigma_{\varepsilon,u}$	$-2.7563 \cdot 10^{-5}$

Table 12: Parameters of the model.

	Data			Model		
Series	s.d (%)	σ_x/σ_{pib}	corr(gdp,x)	s.d (%)	σ_x/σ_{pib}	corr(gdp,x)
GDP	1.129	1	1	1.129	1	1
Total consumption	1.038	0.91	0.79	0.886	0.784	0.794
Domestic consumption	1.016	0.89	0.62	0.962	0.852	0.703
Imported consumption	7.665	6.78	0.48	4.986	4.415	0.305
Imported/domestic ratio	7.262	6.43	0.43	5.457	4.832	0.156
Relative price	2.599	2.30	-0.11	2.591	2.294	-0.156

Table 13: Statistics of the series of the model. H-P filter.

5 Results

Tables 13 and 14 show the model implications on the second order moments of the variables. 1000 simulations were computed with a size of 68, that is the size of observed series. The statistics presented are the means of the statistics of each simulation. The standard deviations of the statistics are the standard deviation of those means.

The model is simulated from an initial situation that has to be as similar as possible to the one in 1981.1. That is, K_0, A_0, p_0 and the ratio of imported goods over domestic goods are chosen so that they coincide with the observed values of that moment. Figure 9 shows the ratio between imported goods and domestic goods produced by the model.

5.1 Income

The volatility of the income produced by the model is equal to the one observed in the data, because the volatility of the income of the model is almost exclusively due to the volatility of A_t. The capital stock gives a little variability to the output as it happens in the data.

Figures 10 and 11 show the impulse response function of the variables of the model. In order to calculate them A_t and p_t were assumed to receive a shock of 1% of the size of its corresponding steady state value. In them we can observe how the income reacts to a shock in the average output of the capital and the relative price. The income reacts to changes in the average rate of return of the capital in a much more stressed way than to changes in the relative price.

We can also see how the income does not go back to the former steady state after a shock, as a consequence of the existence of a unitary root. The same thing happen for all the variables of the model.

5.2 Consumption

The statistics of the consumptions produced by the model are the key element for the confirmation of the empiric validity of the model. The model produces volatilities of the aggregated and domestic consumption that are close to the ones observed, about 80 per cent of the volatility of the income. The volatility of the imported consumption and the domestic-imported goods ratio is four times as high as that of the output (in the data it is a little more than six times as high). The model explains 65% of the volatility of the imported-domestic goods ratio. We have to take into account that it is reasonable that the model may produce volatilities that are a little lower than the ones observed because, actually, there are sources

Standard Deviation(%)								
	Y	$C+p\tilde{C}$	C	$\tilde{C}$	I	$\tilde{C}/C$	A	p
	1.129 (0.002)	0.886 (0.001)	0.962 (0.002)	4.986 (0.008)	3.927 (0.007)	5.457 (0.009)	1.126 (0.002)	2.591 (0.004)
Relative Volatility (σ_x/σ_y)								
	Y	$C+p\tilde{C}$	C	$\tilde{C}$	I	$\tilde{C}/C$	A	p
	1.000	0.784	0.852	4.415	3.477	4.832	0.997	2.294
Observed Relative Volatility								
	Y	$C+p\tilde{C}$	C	$\tilde{C}$	I	$\tilde{C}/C$	A	p
	1	0.91	0.89	6.78	4.58	6.43	0.9067	2.30
Correlations								
	Y	$C+p\tilde{C}$	C	$\tilde{C}$	I	$\tilde{C}/C$	A	p
	1.000 (0.000)	0.794 (0.093)	0.703 (0.124)	0.305 (0.200)	0.781 (0.090)	0.156 (0.214)	0.974 (0.015)	−0.156 (0.214)
	0.794 (0.093)	1.000 (0.000)	0.990 (0.005)	−0.280 (0.212)	0.266 (0.219)	−0.427 (0.191)	0.741 (0.112)	0.427 (0.191)
	0.703 (0.124)	0.990 (0.005)	1.000 (0.000)	−0.407 (0.195)	0.136 (0.231)	−0.546 (0.167)	0.652 (0.140)	0.546 (0.167)
	0.305 (0.200)	−0.280 (0.212)	−0.407 (0.195)	1.000 (0.000)	0.796 (0.088)	0.987 (0.006)	0.332 (0.197)	−0.987 (0.006)
	0.781 (0.090)	0.266 (0.219)	0.136 (0.231)	0.796 (0.088)	1.000 (0.000)	0.701 (0.119)	0.801 (0.085)	−0.701 (0.119)
	0.156 (0.214)	−0.427 (0.191)	−0.546 (0.167)	0.987 (0.006)	0.701 (0.119)	1.000 (0.000)	0.189 (0.213)	−1.000 (0.000)
	0.974 (0.015)	0.741 (0.112)	0.652 (0.140)	0.332 (0.197)	0.801 (0.085)	0.189 (0.213)	1.000 (0.000)	−0.189 (0.213)
	−0.156 (0.214)	0.427 (0.191)	0.546 (0.167)	−0.987 (0.006)	−0.701 (0.119)	−1.000 (0.000)	−0.189 (0.213)	1.000 (0.000)
Cross Correlation between Y(t) and x(t+l)								
	Y	$C+p\tilde{C}$	C	$\tilde{C}$	I	$\tilde{C}/C$	A	p
lag= -5	−0.068 (0.151)	−0.131 (0.173)	−0.132 (0.184)	0.044 (0.223)	0.025 (0.197)	0.063 (0.226)	−0.001 (0.170)	−0.063 (0.226)
lag= -4	0.052 (0.154)	−0.034 (0.176)	−0.045 (0.188)	0.080 (0.223)	0.117 (0.198)	0.081 (0.226)	0.114 (0.170)	−0.081 (0.226)
lag= -3	0.212 (0.154)	0.099 (0.177)	0.074 (0.189)	0.124 (0.220)	0.235 (0.193)	0.100 (0.224)	0.264 (0.167)	−0.100 (0.224)
lag= -2	0.418 (0.141)	0.279 (0.168)	0.237 (0.182)	0.172 (0.216)	0.380 (0.176)	0.116 (0.222)	0.452 (0.149)	−0.116 (0.222)
lag= -1	0.681 (0.093)	0.509 (0.139)	0.445 (0.159)	0.236 (0.209)	0.565 (0.139)	0.138 (0.218)	0.690 (0.098)	−0.138 (0.218)
lag= 0	1.000 (0.000)	0.794 (0.093)	0.703 (0.124)	0.305 (0.200)	0.781 (0.090)	0.156 (0.214)	0.974 (0.015)	−0.156 (0.214)
lag= 1	0.681 (0.093)	0.582 (0.135)	0.523 (0.153)	0.172 (0.209)	0.482 (0.140)	0.065 (0.216)	0.611 (0.097)	−0.065 (0.216)
lag= 2	0.418 (0.141)	0.401 (0.168)	0.368 (0.178)	0.066 (0.210)	0.245 (0.165)	−0.004 (0.214)	0.323 (0.138)	0.004 (0.214)
lag= 3	0.212 (0.154)	0.254 (0.183)	0.240 (0.190)	−0.012 (0.210)	0.065 (0.172)	−0.053 (0.213)	0.106 (0.144)	0.053 (0.213)
lag= 4	0.052 (0.154)	0.135 (0.190)	0.137 (0.198)	−0.070 (0.211)	−0.068 (0.171)	−0.088 (0.215)	−0.055 (0.139)	0.088 (0.215)
lag= 5	−0.068 (0.151)	0.041 (0.195)	0.054 (0.203)	−0.109 (0.209)	−0.162 (0.166)	−0.109 (0.215)	−0.168 (0.135)	0.109 (0.215)

Table 14: Statistics of the cyclical component of the simulated series. H-P filter $\lambda = 1600$, after taking logarithms. In brackets, the standard deviation of the statistic

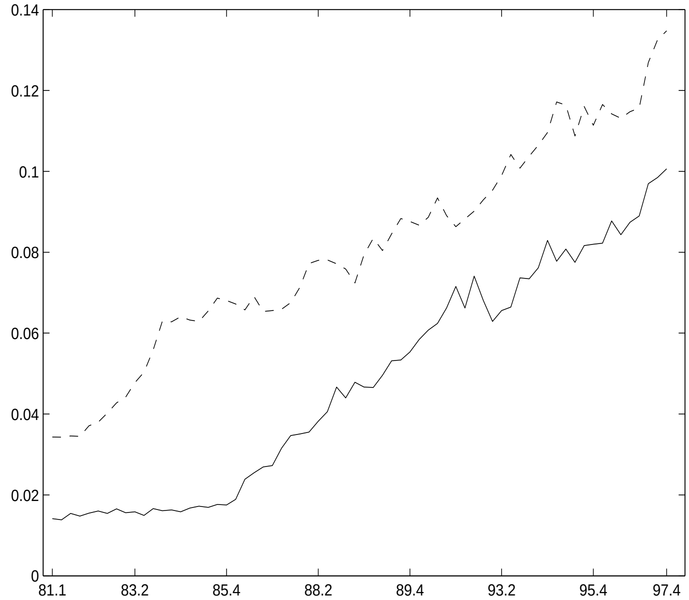


Figure 9: Ratio of the imported and domestic non durable consumer goods produced by the model

of volatility that are not considered by the model, more concretely the labour market. On the other hand, the model also reproduces the correlations of the aggregated and domestic consumption with the output in a proper way.

The impulse response functions show how the consumptions react to a shock in the capital average output in the same way and a little less than the income. This states that a greater incentive to accumulate, due to the higher capital return, partially compensates the incentive to increase consumption, as a result of the income obtained. However, the reaction of the consumptions to a price-increasing shock is different. The domestic consumption increases while the imported one decreases. Consequently the aggregated consumption is increased, because of the greater weight of domestics, but not as much as the latter ones.

5.3 Relative Price

The model generates a correlation between the relative price and the output that has the same sign and the same size as the one observed in the data.

5.4 Investment

Investment is the variable that most reacts to changes in the capital return. The volatility is a little lower than the one in the data like the rest of the variables.

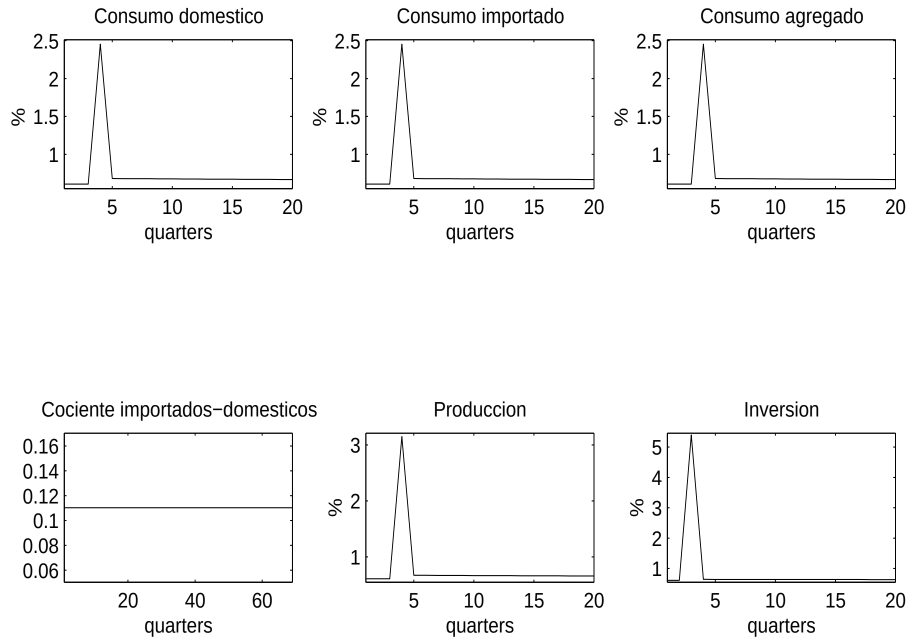


Figure 10: Impulse Response Function of A_t.

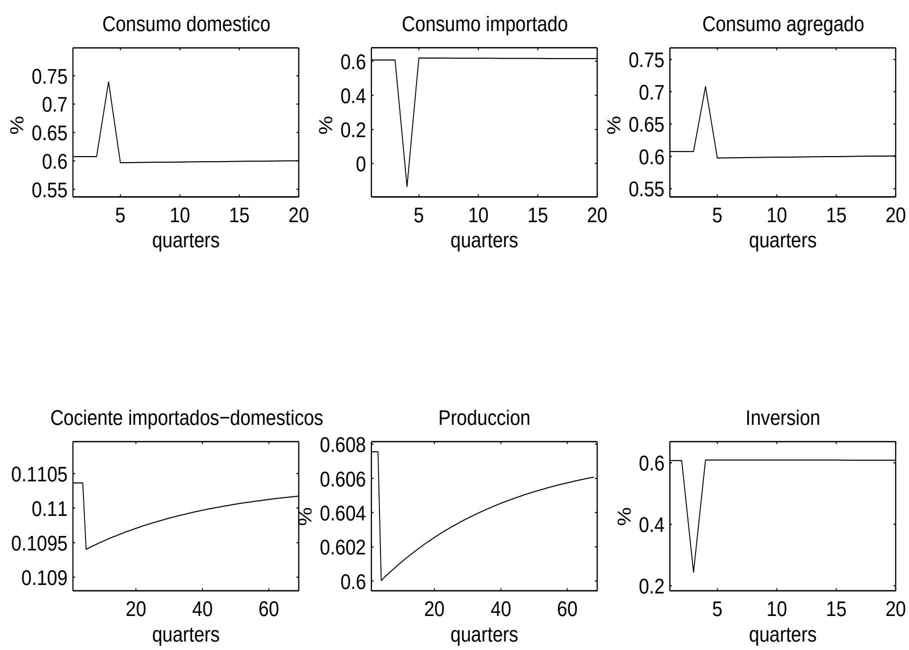


Figure 11: Impulse Response Function of p_t.

	σ_x/σ_y				$corr(x_t, y_t)$			
	$C_t + p_t\tilde{C}_t$	C_t	$\tilde{C}_t$	$\tilde{C}_t/C_t$	$C_t + p_t\tilde{C}_t$	C_t	$\tilde{C}_t$	$\tilde{C}_t/C_t$
Base Model	0.784	0.852	4.415	4.832	0.794	0.703	0.305	0.156
δ	0.9398	1.0016	4.4251	4.8458	0.8209	0.7461	0.3424	0.1600
σ	0.8090	0.8784	4.6435	5.1026	0.7633	0.6782	0.2952	0.1532
γ	0.7975	0.8641	4.4178	4.8318	0.7997	0.7110	0.3084	0.1562
a	0.7274	0.7525	4.6326	4.8602	0.8813	0.8333	0.3061	0.1644
μ	0.7893	0.8568	4.4150	4.8322	0.7950	0.7051	0.3064	0.1562

Table 15: Analysis of sensitivity.

5.5 Persistence

An interesting element in macroeconomics has been the identification of factors that may spread the effect of economic shocks in a period time. It is well known that standard R.B.C. models produce a persistence of the output of equilibrium that follow the characteristics of persistence of the shocks incorporated in the model very closely. The model is not different from the standard R.B.C. models in this aspect. In figure 14 we can observe how the autocorrelations of the income follow the autocorrelations of the capital return very closely.

5.6 Analysis fo Sensitivity

Table 15 supplies information to analyze the effects of a 5% increase in the values of the parameters used in the simulations over the second moments of the variables of the model. The parameter that affects the values of the statistics the most is the discount rate, by increasing the volatility of all the variables and their procyclicity. The other parameters do not affect the value of the statistics very much; therefore we may conclude that the results of the model are robust.

	Data			Model		
Series	s.d (%)	σ_x/σ_{gdp}	corr(gdp,x)	s.d (%)	σ_x/σ_{gdp}	corr(gdp,x)
GDP	1.428	1	1	1.467	1	1
Total Consumption	0.9117	0.63	0.76	0.875	0.596	0.9783
Domestic consumption	0.869	0.60	0.71	0.888	0.605	0.9708
Imported consumption	3.746	2.62	0.58	5.124	3.491	0.0468
Imported/domestic ratio	3.567	2.49	0.44	5.286	3.602	-0.1139
Relative price (AI/DefCND)	3.765	2.63	0.17	3.720	2.535	0.113

Table 16: Statistics of the series of the model for the U.S.A. H-P filter. 1981.1-1997.4.

5.7 Results for the U.S.A.

The model proposed in former sections does not lie in some political or institutional peculiarity of the Spanish economy, therefore, it should be able to explain the behaviour of consumption in other market economies, besides the Spanish one. This section proves the aptitude of the model to reproduce the observed behaviour of the variables in the U.S. economy. Table 17 shows the values of the parameters of the model calibrated for the U.S. economy. Such values were obtained by following the same strategy as in section 4.

Table 16 shows the statistics produced by the model calibrated for the U.S.A. along with the observed corresponding values. In this table we can check how the model reproduces the volatilities of the U.S. total and domestic consumption of non durable goods suitably. On the contrary, the volatilities of the imported consumption and the proportion of imported goods over domestic goods are overvalued by the model in 36 and 48%.

5.8 A Reference Model

In this section the results of the model are compared with a reference model with the same structure but with only one consumer good. The intention is to check that the previous results are due to the imported consumer good with a very volatile relative price and not to the particular structure of the model.

In this model the representative consumer has a utility function with the following shape

$$E\left\{\sum_{t=0}^{\infty}\delta^t\frac{C_t^{1-\gamma}}{1-\gamma}\right\}$$

with $\gamma > 0,\ \gamma \neq 1$ y $0 < \delta < 1$.

The evolution of the level of accumulated assets is given by

$$K_{t+1} = (A_t + 1 - \mu)K_t - C_t$$

where A_t evolves by means of the same equation in stochastic differences as the one in section 3 and $0 < \mu < 1$ is the capital depreciation rate.

Preferences		
Discount Rate	δ	0.9174
Elasticity of Substitution $(1/\sigma - 1)$	σ	0.2962
Elasticity of Intertemporal Substitution$(1/\gamma)$	γ	2.77
Domestic Consumption Weight	a	0.9134
Depreciation Rate	μ	0.012
Stochastic Processes		
Constant of the process of capital return	$\overline{A}$	-0.0821
Capital return autocorrelation coefficient	ϕ_1	0.9612
Standard deviation of shock of capital return	σ_ε	0.0120
Constant of the process of the relative price	$\overline{p}$	-0.0017
Relative price autocorrelation coefficient	θ_1	0.9325
Standard deviation of shock of relative price	σ_u	0.0298
Covariance between shocks	$\sigma_{\varepsilon,u}$	0.042010^{-3}

Table 17: Parameters of the model calibrated for the U.S.A.

In this case the problem of intertemporal choice of the representative agent is

$$\max_{C_t} E\left\{\sum_{t=0}^{\infty} \delta^t \frac{C_t^{1-\gamma}}{1-\gamma}\right\} \tag{4}$$

s.t:

$$C_t = (A_t + 1 - \mu)K_t - K_{t+1}$$
$$A_{t+1} = \phi(A_t, \varepsilon_{t+1})$$

Two experiments are carried out with this model: one with A_t behaving as an *i.i.d* process and another with A_t following the same process as in the model with two goods. The details of the solution of this model are in the appendix.

5.8.1 Results when A_t is *i.i.d*

In this case it is assumed that

$$\log(A_t) = \overline{A} + \varepsilon_t$$

where A_t includes the term of depreciation. The solution of the model (4) can be calculated explicitly

$$K_{t+1} = \left(\delta E\left[A_t^{1-\gamma}\right]\right)^{\frac{1}{\gamma}} A_t K_t$$
$$C_t = \left[1 - \left(\delta E\left[A_t^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}\right] A_t K_t$$
$$I_t = \left[\left(\delta E\left[A_t^{1-\gamma}\right]\right)^{\frac{1}{\gamma}} A_t - 1 + \mu\right] K_t$$
$$Y_t = A_t K_t$$

Note that the optimum consumption is a linear function of the income, so that the growth

Preferences		
Discount Rate	δ	0.9368
Elasticity of intertemporal substitution$(1/\gamma)$	γ	2.77
Depreciation rate	μ	0.0147
Stochastic processes		
Constant of the process of capital return	$\overline{A}$	0.07243
Standard deviation of shock of capital return	σ_ε	0.0026

Table 18: Values of the parameters of the reference model with A_t *i.i.d.*

	Data			**Model**		
Serie	s.d (%)	σ_x/σ_{gdp}	corr(gdp,x)	s.d (%)	σ_x/σ_{gdp}	corr(gdp,x)
Income	1.129	1	1	1.1330	1	1
Consumption	1.038	0.91	0.79	1.1330	1	1
Investment	5.170	4.58	0.82	0.8897	0.7853	0.4010

Table 19: Statistics of the series of the reference model with A_t *i.i.d.* H-P Filter. 1981.1-1997.4.

rates of the consumption and income will be the same ones

$$\frac{C_{t+1}}{C_t} = \frac{Y_{t+1}}{Y_t} = \left(\delta E[A_t^{1-\gamma}]\right)^{\frac{1}{\gamma}} A_t$$

that is, the consumer reacts to a non-expected increase of the income by shifting that increase to the consumption. The reason is that since A_t is *i.i.d* a positive shock today does not give any information about future shocks, so the best thing the consumer can do is to shift that non expected increase to his consumption; so that in this case the volatility of the consumption is equal to the volatility of the income.

The value of $\overline{A}$ is chosen to be the observed mean of $\log((\text{GDP}_{\text{pm}}/K_t) + 1 - \mu)$ for the Spanish economy in the 1981.1-1997.4 period, and σ_ε^2 its variance. In this case δ is chosen so that

$$\left[\delta\left(e^{\overline{A}}\right)\right]^{\frac{1}{\gamma}} = 1.0029$$

The rest of the parameters are the same ones as in the model with two goods. See table 18 for these values.

Finally, table 19 shows the statistics of the deviations of the trend of the series generated by the model, obtained by means of H-P filter with $\lambda = 1600$. The same number of simulations were carried out with the same size as for the series of the model with two goods.

5.8.2 Results with A_t autocorrelated

In this case an evolution of A_t is considered to be like that of the model with two goods, that is, the evolution of A_t is given by

$$A_{t+1} = e^{\overline{A}} A_t^{\phi} e^{\varepsilon_{t+1}}$$

Preferences		
Discount Rate	δ	0.9368
Elasticity of intertemporal substitution ($1/\gamma$)	γ	2.77
Depreciation Rate	μ	0.0147
Stochastic Processes		
Constant of the process of the capital return	$\overline{A}$	-0.0343
Capital return autocorrelation coefficient	ϕ_1	0.9857
Standard deviation of shock of capital return	σ_ε	0.0052

Table 20: Values of the parameters of the reference model with A_t autocorrelated.

	Data			**Model**		
Series	s.d (%)	σ_x/σ_{pib}	corr(gdp,x)	s.d (%)	σ_x/σ_{pib}	corr(gdp,x)
Income	1.129	1	1	1.1201	1	1
Consumption	1.038	0.91	0.79	0.7781	0.6947	0.9978
Investment	5.1708	4.58	0.82	2.5596	2.2852	0.9959

Table 21: Statistics of the series of the reference model with At autocorrelated. H-P filter 1981.1-1997.4.

The values of the parameters used are presented in table 20

Table 21 shows how the consumption series simulated with the reference model present a much lower volatility than the volatility of the income. Unlike the case in which A_t evolves as $i.i.d$, in this case a positive shock today causes that the rate of return of capital is high and gives information about the future realization of this rate of return, because if it is high today, with high probability, it will also be high in the following period. This implies an incentive to accumulate, so that the incentive to increase consumption as a result of a higher income obtained in the period is partially compensated. Thus, the volatility of consumption is much lower than the volatility of the income.

6 Conclusion

The excessive volatility of the Spanish aggregated consumption with relation to the aggregated income has been stated in this work. We can even observe this high volatility when the consumption of durable goods is not taken into account. It has also been reported that this is not an exclusive phenomenon of the Spanish economy. Other European countries also show this behaviour of the data of consumption. The majority view suggested by R.B.C. literature for the Spanish economy is that this fact is either a strong sign of the lack of instruments to smoothen consumption by means of the credit market or that the existence of frequent changes in the tax and transfer patterns are responsible for this behaviour. However, the observation that the Spanish economy is an open one and is exposed to strong variations in its real exchange rate suggests that the former answers are not so obvious. In fact, this work makes evident that a model of intertemporal optimization where there is no credit constraint or government can produce so high volatilities of consumption as the ones observed. The excess of volatility of consumption can not be seen, per se, as an evidence of lack of opportunities to smoothen consumption or of intertemporal rationality.

Appendices

A Solving the Model of the Section 3

The F.O.C. of the problem (1) are

$$\frac{1-a}{a}\left(\frac{\tilde{C}_t}{C_t}\right)^{\sigma-1} - p_t = 0 \tag{5}$$

$$\left[aC_t^{\sigma} + (1-a)\tilde{C}_t^{\sigma}\right]^{\frac{1-\gamma-\sigma}{\sigma}} C_t^{\sigma-1} - \delta E_t\left[(A_{t+1}+1-\mu)\left[aC_{t+1}^{\sigma} + (1-a)\tilde{C}_{t+1}^{\sigma}\right]^{\frac{1-\gamma-\sigma}{\sigma}} C_{t+1}^{\sigma-1}\right] = 0$$

$$C_t + p_t\tilde{C}_t = (A_t + 1 - \mu)K_t - K_{t+1}$$

$$A_{t+1} = \phi(A_t, \varepsilon_{t+1})$$

$$p_{t+1} = \theta(p_t, u_{t+1})$$

by substituting the expression (5) in the rest of them the F.O.C. can be written as

$$\begin{aligned}
&\left[a + (1-a)\left(p_t\frac{a}{1-a}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1-\gamma-\sigma}{\sigma}} C_t^{-\gamma} - \\
&-\delta E_t\left[(A_{t+1}+1-\mu)\left[a + (1-a)\left(p_{t+1}\frac{a}{1-a}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1-\gamma-\sigma}{\sigma}} C_{t+1}^{-\gamma}\right] = 0
\end{aligned} \tag{6}$$

$$C_t\left[1 + p_t\left(p_t\frac{a}{1-a}\right)^{\frac{1}{\sigma-1}}\right] = (A_t+1-\mu)K_t - K_{t+1}$$

$$A_{t+1} = \phi(A_t, \varepsilon_{t+1})$$

$$p_{t+1} = \theta(p_t, u_{t+1})$$

finally by calling $c_t = C_t/K_t$ and $\lambda_{t+1} = K_{t+1}/K_t$ we have

$$\begin{aligned}
&\left[a + (1-a)\left(p_t\frac{a}{1-a}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1-\gamma-\sigma}{\sigma}} c_t^{-\gamma} - \\
&-\delta E_t\left[(A_{t+1}+1-\mu)\left[a + (1-a)\left(p_{t+1}\frac{a}{1-a}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1-\gamma-\sigma}{\sigma}} c_{t+1}^{-\gamma}\lambda_{t+1}^{-\gamma}\right] = 0
\end{aligned} \tag{7}$$

$$c_t\left[1 + p_t\left(p_t\frac{a}{1-a}\right)^{\frac{1}{\sigma-1}}\right] = (A_t+1-\mu) - \lambda_{t+1} \tag{8}$$

$$A_{t+1} = \phi(A_t, \varepsilon_{t+1}) \tag{9}$$

$$p_{t+1} = \theta(p_t, u_{t+1}) \tag{10}$$

A.1 The Solution of the Determinist Case

In absence of uncertainty the solution of the optimization problem leads to a balanced growth path with a constant growth rate for Y, C, $\tilde{C}$ and K. In the determinist case, $\varepsilon_t = u_t = 0$

and the F.O.C. are

$$
\begin{aligned}
&-\left(c_t\right)^{-\gamma}+\delta(A+1-\mu)\left(c_{t+1}\right)^{-\gamma}\lambda_{t+1}^{-\gamma}=0\\
&c_t\left[1+p\left(p\frac{a}{1-a}\right)^{\frac{1}{\sigma-1}}\right]=(A+1-\mu)-\lambda_{t+1}\\
&A=\phi(A,0)\;\; p=\theta(p,0)
\end{aligned}
$$

From where

$$
\lambda_{t+1}=f(\lambda_t)=(A+1-\mu)-(\delta(A+1-\mu))^{\frac{1}{\gamma}}\left[\frac{A+1-\mu}{\lambda_t}-1\right] \tag{11}
$$

taking into account that

$$
\begin{aligned}
&\lim_{\lambda_t\to\infty} f(\lambda_t)=(A+1-\mu)+(\delta(A+1-\mu))^{\frac{1}{\gamma}} \quad \lim_{\lambda_t\to 0} f(\lambda_t)=-\infty\\
&f'(\lambda_t)=\frac{(A+1-\mu)\,(\delta(A+1-\mu))^{\frac{1}{\gamma}}}{\left(\lambda_t\right)^2}>0\\
&f''(\lambda_t)=-\frac{(A+1-\mu)\,(\delta(A+1-\mu))^{\frac{1}{\gamma}}\,2}{\left(\lambda_t\right)^3}<0 \text{ para } \lambda_t>0
\end{aligned}
$$

and that the fixed points $\lambda=f(\lambda)$ are

$$
\lambda_1=(\delta(A+1-\mu))^{\frac{1}{\gamma}} \quad \lambda_2=A+1-\mu
$$

we conclude that the solution of (11) what not imply $c_t\leq 0$ in some t is $\lambda_t=\lambda_1$. The behaviour of the solution of (11) can be seen in figure 12.

Thus, the solution is

$$
\lambda_t=(\delta(A+1-\mu))^{\frac{1}{\gamma}} \quad c_t=\frac{(A+1-\mu)-(\delta(A+1-\mu))^{-\frac{1}{\gamma}}}{\left[1+p\left(p\frac{a}{1-a}\right)^{\frac{1}{\sigma-1}}\right]} \text{ with } \left(\delta(A+1-\mu)^{1-\gamma}\right)^{\frac{1}{\gamma}}<1
$$

A.2 Solving the Stochastic Case

The stochastic case does not have an explicit solution, so it is necessary to approximate the solution. If we take into account that at the beginning of each period it is enough to know the value of A_t and p_t to describe the state of the system given by (7)-(10), we can write $c_t=c(A_t,p_t)$. The strategy followed is to approximate the function $c(\cdot)$ by means of a weighted sum of polynomials and to use projection methods to calculate the coefficients of the weights. A complete description of these techniques can be found in Judd (1998).

Then, the system (7)-(10) can be written

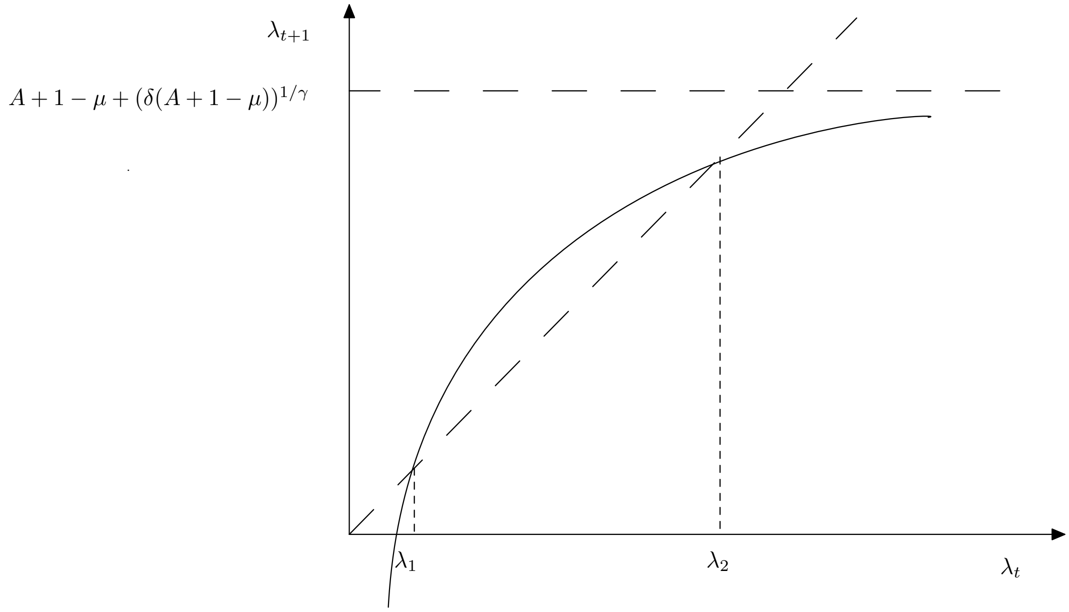


Figure 12: The solution of (11).

$$
\begin{aligned}
&\left[a+(1-a)\left(p\frac{a}{1-a}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1-\gamma-\sigma}{\sigma}}[c(A,p)]^{-\gamma}-\\
&-\delta E\left[(A'+1-\mu)\left[a+(1-a)\left(p'\frac{a}{1-a}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1-\gamma-\sigma}{\sigma}}\left[c(A',p')\right]^{-\gamma}(\lambda)^{-\gamma}\mid A,p\right]=0\\
&\lambda=(A+1-\mu)-c(A',p')\left[1+p'\left(p'\frac{a}{1-a}\right)^{\frac{1}{\sigma-1}}\right]\\
&A'=\phi(A,\varepsilon')\\
&p'=\theta(p,u')
\end{aligned}
$$

The former system is solved numerically by approximating the function $c(A,p)$

$$\hat{c}(A,p;a)=\sum_{i=1}^{n_A}\sum_{h=1}^{n_p}a_{ih}\psi_{ih}(A,p)$$

where

$$\psi_{ih}(A,p)=T_{i-1}\left(\frac{2(A-A_m)}{A_M-A_m}-1\right)T_{h-1}\left(\frac{2(p-p_m)}{p_M-p_m}-1\right)$$

with $T_{i-1}(x)$ equal to the Chebyshev polynomial[4] of degree $i-1$ valued in x. The points

[4] $T_i(x)=\cos(i\cos^{-1}(x))$ and we can obtain it recursively with $T_0(x)=1$, $T_1(x)=x$ y $T_{i+1}(x)=2xT_i(x)-T_{i-1}(x)$.

(A, p) are chosen such that $(A, p) \in [A_m, A_M] \times [p_m, p_M]$ and are

$$A_{l_A} = A_m + \frac{1}{2}(A_M - A_m)(x_{l_A}^{n_A} + 1) \;\; l_A = 1, \ldots, n_A$$
$$p_{l_p} = p_m + \frac{1}{2}(p_M - p_m)(x_{l_p}^{n_p} + 1) \;\; l_p = 1, \ldots, n_p$$
$$x_l^n = \cos(\frac{(2l-1)\pi}{2n}) \;\; l = 1, \ldots, n \quad n = n_A, n_p.$$

where x_l^n are the zeros of the Chebyshev polynomial of degree n, and the extremes of the rectangle are chosen by truncating the innovations $\varepsilon \in [-3\sigma_\varepsilon, 3\sigma_\varepsilon]$ and $u \in [-3\sigma_u, 3\sigma_u]$ so that A and p are confined to $[A_m, A_M]$ and $[p_m, p_M]$, taking into account that the evolution of A_t and p_t is supposed to be given by

$$\begin{pmatrix} \log(A_{t+1}) \\ \log(p_{t+1}) \end{pmatrix} = \begin{pmatrix} \overline{A} \\ \overline{p} \end{pmatrix} + \begin{pmatrix} \phi_1 & 0 \\ 0 & \theta_2 \end{pmatrix} \begin{pmatrix} \log(A_t) \\ \log(p_t) \end{pmatrix} + \begin{pmatrix} \varepsilon_{t+1} \\ u_{t+1} \end{pmatrix} \tag{12}$$

the extremes of the intervals are calculated in the following way

$$\begin{pmatrix} \log(A_m) \\ \log(p_m) \end{pmatrix} = \left[I - \begin{pmatrix} \phi_1 & 0 \\ 0 & \theta_2 \end{pmatrix} \right]^{-1} \begin{pmatrix} \overline{A} + \varepsilon_m \\ \overline{p} + u_m \end{pmatrix}$$
$$\begin{pmatrix} \log(A_M) \\ \log(p_M) \end{pmatrix} = \left[I - \begin{pmatrix} \phi_1 & 0 \\ 0 & \theta_2 \end{pmatrix} \right]^{-1} \begin{pmatrix} \overline{A} + \varepsilon_M \\ \overline{p} + u_M \end{pmatrix}$$

So, we have to calculate $a \in \mathbb{R}^{n_A n_p}$ that satisfies

$$\mathcal{R}(A, p; a) = \hat{c}(A, p; a) - \left[\frac{A + 1 - \mu - \left[1 + p \left(p \frac{a}{1-a}\right)^{\frac{1}{\sigma - 1}}\right] \hat{c}(A, p; a)}{\left[a + (1-a)\left(p\frac{a}{1-a}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\gamma+\sigma-1}{\gamma\sigma}}} \right.$$
$$\left. \left(\delta E\left[h(y_1, y_2; a)\right]\right)^{-\frac{1}{\gamma}} \right] = 0 \tag{13}$$

where

$$h(y_1, y_2; a) = (1 - \mu + e^{\overline{A}} A^{\phi_1} e^{y_1}) \left[a + (1-a) \left(e^{\overline{p}} p^{\theta_2} e^{y_2} \frac{a}{1-a} \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{1-\gamma-\sigma}{\sigma}}$$
$$\left(\hat{c}(e^{\overline{A}} A^{\phi_1} e^{y_1}, e^{\overline{p}} p^{\theta_2} e^{y_2}; a) \right)^{-\gamma}$$

Given the assumptions about the stochastic processes that govern A and p, we can write that $\log(A')$, conditional to $\log(A)$, is distributed as $\overline{A} + \phi_1 \log(A) + y_1$ and that $\log(p')$,conditional to $\log(p)$, is distributed as $\overline{p} + \theta_2 \log(p) + y_2$ with $y = (y_1, y_2)^\top \sim N\left((0,0)^\top, \Sigma\right)$, so that

$$E\left\{h(y_1, y_2; a)\right\} = \int_{\mathbb{R}^2} h(y; a)(2\pi)^{-1}|\Sigma|^{-1/2} e^{-\frac{1}{2}y^\top \Sigma^{-1} y} dy =$$

the matrix Σ admits a Cholesky decomposition such that $\Sigma = \Omega\Omega^\top$ so that $\Sigma^{-1} = (\Omega^{-1})^\top \Omega^{-1}$ and making the change of variable $x = \Omega^{-1} y / \sqrt{2} \Rightarrow y = \sqrt{2}\Omega x$ we have

$$= \pi^{-1}|\Sigma|^{-1/2} \int_{\mathbb{R}^2} h(\sqrt{2}\Omega x; a) e^{-\sum_{i=1}^2 x_i^2} |\det \Omega| dx$$

this integral can be approximated by Gauss-Hermite quadrature, so that

$$E\left\{h(y_1, y_2; a)\right\} \approx \sum_{i=1}^{m_z}\sum_{h=1}^{m_z} \varpi_i \varpi_h h(\sqrt{2}\Omega[x_i\ x_h]^\top; a)\pi^{-1}|\Sigma|^{-1/2}|\det\Omega|$$

where ϖ y x are the Gauss-Hermite weights and nodes for m_z. Finally we just have to solve the system given by

$$\mathcal{R}(A_i, p_h; a) = 0 \quad i = 1, \ldots, n_A \quad h = 1, \ldots, n_p$$

This is a non-linear system in a. To find a any method can be used to solve non-linear equations. In this case, we use the Galerkin method that implies to solve

$$P_{ih}(a) = \sum_{l_A=1}^{n_A}\sum_{l_p=1}^{n_p} \mathcal{R}(A_{l_A}, p_{l_p}; a)\psi_{ih}(A_{l_A}, p_{l_p}) = 0$$
$$i = 1, \ldots, n_A \quad h = 1, \ldots, n_p$$

B Solving the model of the section 5.8

B.1 A_t *i.i.d*

We neglect the term $(1-\mu)$ of the problem (4) to simplify the exposition. So, A_t refers to the variable net of depreciation.

We can consider the case when the evolution of A_t is

$$\log(A_t) = \overline{A} + \varepsilon_t$$

In this case the FOC are

$$\begin{aligned}
&-u'(C_t) + \delta\, E\left\{A_{t+1}u'(C_{t+1})\right\} = 0\\
&C_t = A_t K_t - K_{t+1}\\
&A_t = e^{\overline{A}} e^{\varepsilon_t}\\
&\lim_{t\to\infty} \delta^t E\left[u'(C_t)A_t K_t\right] = 0
\end{aligned}$$

The last problem can be writing

$$V(\varepsilon, K) = \max_{0\le K' \le AK}\left\{u(AK - K') + \delta\int V(\varepsilon', K')\mu(d\varepsilon')\right\} \tag{14}$$

where $u(C) = C^{1-\gamma}/1-\gamma$ with $\gamma > 0,\ \gamma \neq 1$.

Proposition 1. *If* $\delta E\left[A_t^{1-\gamma}\right] < 1$, *the value function* V *exists, is unique and continuous. Moreover, there is an optimal feasible plan which verifies* (14).

Proof: To proof the proposition is enough to find a function $\psi : \mathcal{R}\times\mathcal{R}_+ \to \mathcal{R}_+$ and $\alpha \in (0,1)$ which verify:

a.- $u(C) \le \psi(\varepsilon, K)$ for all $(\varepsilon, K) \in \mathcal{R}\times\mathcal{R}_+$ and C feasible.

b.-

$$\delta \sup_{0\leq K'\leq AK} \frac{\int \psi(s,K')\eta(ds)}{\psi(\varepsilon,K)} \leq \alpha$$

where η is the Normal probability measure on $\mathcal{R}$. In this case, the function $\psi(\varepsilon,K) = (1-\gamma)u(AK)$ is considered and take into account that

$$\int u(A'K')\eta(ds) \leq u\left(K'\right)\int u\left(A'\right)\eta(ds) \leq u\left(AK\right)\int u\left(A'\right)\eta(ds)$$

where $A' = e^{\overline{A}+s}$. ψ verifies condition a, and

$$\delta \sup_{0\leq K'\leq AK} \frac{\int \psi(s,K')\eta(ds)}{\psi(\varepsilon,K)} \leq \delta E\left[A_t^{1-\gamma}\right]$$

Then, with $\alpha = \delta E\left[A_t^{1-\gamma}\right]$ by theorem 1 in Durán (2000) follows the proposition. □

Proposition 1 says that, in order to the problem be well defined, it is necessary to discount the expect feasible growth.

B.1.1 The Solution of the Determinist Case

In absence of uncertainty the solution of the optimization problem leads to a balanced growth path with a constant growth rate for Y, C, and K. In the determinist case, $\varepsilon_t = 0$ and the F.O.C. are

$$\begin{aligned} &-u'\left(C_t\right) + \delta A u'\left(C_{t+1}\right) = 0 \\ &C_t = AK_t - K_{t+1} \\ &\lim_{t\to\infty} \delta^t u'(C_t)AK_t = 0 \end{aligned}$$

with $A = \exp(\overline{A})$. The former system can be written as

$$\begin{aligned} &-\left(c_t\right)^{-\gamma} + \delta A\left(c_{t+1}\right)^{-\gamma}\lambda_{t+1}^{-\gamma} = 0 \\ &c_t = A - \lambda_{t+1} \\ &\lim_{t\to\infty} \delta^t u'(c_t)A = 0 \end{aligned}$$

con $c_t = C_t/K_t$ y $\lambda_{t+1} = K_{t+1}/K_t$. From where

$$\lambda_{t+1} = f(\lambda_t) = A - (\delta A)^{\frac{1}{\gamma}}\left[\frac{A}{\lambda_t} - 1\right] \tag{15}$$

take into account that

$$\lim_{\lambda_t\to\infty} f(\lambda_t) = A + (\delta A)^{\frac{1}{\gamma}} \quad \lim_{\lambda_t\to 0} f(\lambda_t) = -\infty$$

$$f'(\lambda_t) = \frac{A\left(\delta A\right)^{\frac{1}{\gamma}}}{\left(\lambda_t\right)^2} > 0 \;\; f''(\lambda_t) = -\frac{A\left(\delta A\right)^{\frac{1}{\gamma}}2}{\left(\lambda_t\right)^3} < 0 \text{ para } \lambda_t > 0$$

and that the fixed points $\lambda = f(\lambda)$ are

$$\lambda_1 = (\delta A)^{\frac{1}{\gamma}} \quad \lambda_2 = A$$

we conclude that the solution of (15) which verifies the transversality condition is $\lambda_t = \lambda_1$. The behaviour of the solution of (15) can be seen in figure 13.

Thus, the solution is

$$K_{t+1} = (\delta A)^{\frac{1}{\gamma}} K_t = \left(\delta A^{1-\gamma}\right)^{\frac{1}{\gamma}} AK_t$$

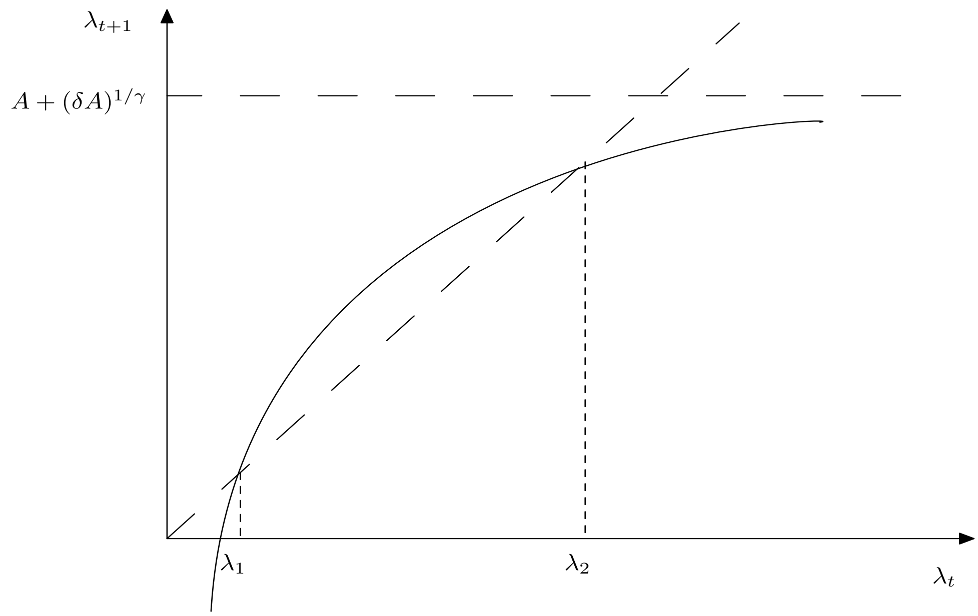


Figure 13: La solución de (15).

B.1.2 Solving the Stochastic Case

We can conjecture that the solution of the stochastic problem is

$$K_{t+1} = \left(\delta E\left[A_t^{1-\gamma}\right]\right)^{\frac{1}{\gamma}} A_t K_t \tag{16}$$

and we will proof that verifies the FOC

$$C_t = \left[1 - \left(\delta E\left[A_t^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}\right] A_t K_t$$
$$C_{t+1} = \left[1 - \left(\delta E\left[A_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}\right] A_{t+1} \left(\delta E\left[A_t^{1-\gamma}\right]\right)^{\frac{1}{\gamma}} A_t K_t$$
$$C_{t+1}^{-\gamma} A_{t+1} = A_{t+1}^{1-\gamma} \left[1 - \left(\delta E\left[A_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}\right]^{-\gamma} \left(\delta E\left[A_t^{1-\gamma}\right]\right)^{-1} (A_t K_t)^{-\gamma}$$

take into account that $E\left[A_t^{1-\gamma}\right] = E\left[A_{t+1}^{1-\gamma}\right]$ from A_t $i.i.d$

$$\delta\, E\left[A_{t+1} C_{t+1}^{-\gamma}\right] = \left[1 - \left(\delta E\left[A_t^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}\right]^{-\gamma} (A_t K_t)^{-\gamma} = C_t^{-\gamma}$$

On the other hand, the transversality condition says

$$\lim_{t\to\infty} \delta^t E\left[u'(C_t) A_t K_t\right] = \lim_{t\to\infty} \delta^t E\left[C_t^{-\gamma} A_t K_t\right]$$

if the policy is given by (16) we have

$$K_t = \alpha^t K_0 \prod_{j=1}^{t} A_{t-j}$$

$$E\left[C_t^{-\gamma}A_tK_t\right] = E\left[\beta\left(A_tK_t\right)^{1-\gamma}\right] = \beta\alpha^{(1-\gamma)t}K_0^{1-\gamma}\prod_{j=1}^{t}E[A_{t-j}^{1-\gamma}] = \beta\alpha^{(1-\gamma)t}K_0^{1-\gamma}(E[A_t^{1-\gamma}])^t$$

where the last two equalities follow from that $\{A_t\}_{t=1}^{\infty}$ is *i.i.d.* and

$$\alpha = (\delta E[A_t^{1-\gamma}])^{\frac{1}{\gamma}} \text{ y } \beta = \left[1-\left(\delta E\left[A_t^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}\right]^{-\gamma}$$

thus, the transversality condition says

$$\lim_{t\to\infty}\delta^t E\left[u'(C_t)A_tK_t\right] = \lim_{t\to\infty}\beta K_0^{1-\gamma}\left(\delta\alpha^{1-\gamma}E[A_t^{1-\gamma}]\right)^t = 0 \Leftrightarrow \delta\alpha^{1-\gamma}E[A_t^{1-\gamma}] < 1 \Leftrightarrow$$
$$\Leftrightarrow \delta E[A_t^{1-\gamma}] < 1$$

In absence of uncertainty this condition is the same condition in proposition 1, $\delta A^{1-\gamma} < 1$.

B.1.3 Long-Run Stochastic Growth Conditions

There is possible to find conditions of long-run growth in this model. The optimal policy function given by (16) implies a stochastic process for the optimal income given by:

$$Y_t = \alpha A_t Y_{t-1}$$

The following lemma helps to characterize the stochastic process of Y_t.

Lemma 1. *Given a sequence of i.i.d. random variables $\{r_t\}$ with support in $\mathcal{R}_+$, and for any $y > 0$ let $\{z_t\}$ a stochastic process define by:*

$$z_t = kr_tz_{t-1},\ t \geq 0, z_0 = y, k > 0$$

Then, $z_t \to +\infty$ almost surely if $E[\ln(kr_t)] > 0$. If $E[\ln(kr_t)] < 0$, then $z_t \to 0$ almost surely.

Proof: One can rewrite z_t as:

$$z_t = z_0k^t\prod_{i=1}^{t}r_i \Rightarrow \ln(z_t) = \ln(z_0) + t\ln(k) + \sum_{i=1}^{t}\ln(r_i) =$$
$$= \ln(z_0) + \sum_{i=1}^{t}\ln(kr_i) = \ln(z_0) + \frac{1}{t}t\sum_{i=1}^{t}\ln(kr_i) \Rightarrow$$
$$\Rightarrow z_t = y\left[e^{\frac{1}{t}\sum_{i=1}^{t}\ln(kr_i)}\right]^t$$

and use the law of large numbers to obtain:

$$\frac{1}{t}\sum_{i=1}^{t}\ln(kr_i) \xrightarrow{c.s} E[\ln(kr_t)]$$

from where the results follows. □

Using the lemma 1 we have that $Y_t \to +\infty$ almost surely if

$$E[\ln(\alpha A_t)] > 0 \Rightarrow E\left[\ln\left((\delta E[A_t^{1-\gamma}])^{\frac{1}{\gamma}}A_t\right)\right] > 0 \Leftrightarrow \gamma E[\ln(A_t)] + \ln(\delta) + \ln(E[A_t^{1-\gamma}]) > 0 \tag{17}$$

In absence of uncertainty this condition is $\delta A > 1$.

In this case is easy to find the expression of $E\left[A_t^{1-\gamma}\right]$ because A_t is distributed as a lognormal

$$E\left[A_t^{1-\gamma}\right] = e^{(1-\gamma)\overline{A}+\frac{1}{2}(1-\gamma)^2\sigma_\varepsilon^2}$$

thus, we can written the condition (17) as

$$\overline{A} + \ln(\delta) + \frac{1}{2}(1-\gamma)^2\sigma_\varepsilon^2 > 0$$

B.2 A_t autocorrelated

The FOC in this case are

$$\begin{aligned}
&-u'\left(C_t\right) + \delta\, E_t\left\{(A_{t+1}+1-\mu)u'\left(C_{t+1}\right)\right\} = 0\\
&C_t = (A_t+1-\mu)K_t - K_{t+1}\\
&A_{t+1} = e^{\overline{A}}A_t^{\phi}e^{\varepsilon_{t+1}}\\
&\lim_{t\to\infty}\delta^t E\left[u'(C_t)(A_t+1-\mu)K_t\right] = 0
\end{aligned}$$

with $u'(C) = C^{-\gamma}$ the system can rewrite as

$$\begin{aligned}
&-u'\left(c_t\right) + \delta\, E\left\{(A_{t+1}+1-\mu)u'\left(c_{t+1}\lambda_{t+1}\right) \mid A_t\right\} = 0\\
&c_t = (A_t+1-\mu) - \lambda_{t+1}\\
&\log(A_{t+1}) = \overline{A} + \phi\log(A_t) + \varepsilon_{t+1}\\
&\lim_{t\to\infty}\delta^t E\left[u'(c_t)(A_t+1-\mu)\right] = 0
\end{aligned}$$

where $c_t = C_t/K_t$ and $\lambda_{t+1} = K_{t+1}/K_t$.

Take into account that at the beginning of each period it is enough to know the value of A_t to describe the state of the system, we can write $c_t = c(A_t)$. Thus we can rewrite the FOC as

$$\begin{aligned}
&-u'\left(c(A)\right) + \delta\, E\left\{(A'+1-\mu)u'\left(c(A')\lambda\right) \mid A\right\} = 0\\
&\lambda = A+1-\mu-c(A)
\end{aligned}$$

with $\log(A')|\log(A) \sim N(\overline{A}+\phi\log(A), \sigma_\varepsilon^2)$.

The former system is solved numerically by approximating the function $c(A)$

$$\hat{c}(A;a) = \sum_{i=1}^{n_A} a_i\psi_i(A)$$

where

$$\psi_i(A) = T_{i-1}\left(\frac{2(A-A_m)}{A_M-A_m} - 1\right)$$

with $T_{i-1}(x)$ equal to the Chebyshev polynomial of degree $i-1$ valued in x. The points A are chosen such that $A \in [A_m, A_M]$ and are

$$\begin{aligned}
&A_{l_A} = A_m + \frac{1}{2}(A_M-A_m)(x_{l_A}+1) \quad l_A = 1,\ldots,n_A\\
&x_{l_A} = \cos\left(\frac{(2l-1)\pi}{2n_A}\right) \quad l = 1,\ldots,n_A
\end{aligned}$$

where x_{l_A} are the zeros of the Chebyshev polynomial of degree n_A, and the extremes of the rectangle are chosen by truncating the innovations $\varepsilon \in [-3\sigma_\varepsilon, 3\sigma_\varepsilon]$ so that A is confined to $[A_m, A_M]$ with $\log(A_m) = (\overline{A} + \varepsilon_m)(1-\phi)^{-1}$, $\log(A_M) = (\overline{A} + \varepsilon_M)(1-\phi)^{-1}$.

So, we have to calculate $a \in \mathcal{R}^{n_A}$ that satisfies

$$0 = \hat{c}(A; a) - [(A + 1 - \mu) - \hat{c}(A; a)] \left[\delta E\left\{(A' + 1 - \mu)\left(\hat{c}(A'; a)\right)^{-\gamma} \mid A\right\}\right]^{-\frac{1}{\gamma}} \tag{18}$$

Given the assumptions about the stochastic process that govern A, we can write that $\log(A')$, conditional to $\log(A)$, is distributed as $\overline{A} + \phi\log(A) + \sigma_\varepsilon y$ with $y \sim N(0,1)$, so that (18) is

$$0 = \hat{c}(A; a) - [(A + 1 - \mu) - \hat{c}(A; a)] [\delta E[h(y; a)]]^{-\frac{1}{\gamma}}$$

where

$$h(y; a) = \left(1 - \mu + e^{\overline{A}} A^\phi e^{\sigma_\varepsilon y}\right) \left[\hat{c}\left(e^{\overline{A}} A^\phi e^{\sigma_\varepsilon y}; a\right)\right]^{-\gamma}$$

On the other hand ,

$$E\{h(y; a)\} = \int_{-\infty}^{\infty} h(y; a)(2\pi)^{-1} e^{-\frac{y^2}{2}} dy =$$

making the change of variable $x = y/\sqrt{2}$ we have

$$= \int_{-\infty}^{\infty} h(\sqrt{2}x; a)\pi^{-1} e^{-x^2} dx$$

this integral can be approximated by Gauss-Hermite quadrature, so that

$$E\{h(y; a)\} \approx \sum_{i=1}^{m_z} \varpi_i h(\sqrt{2}x_i; a)\pi^{-1}$$

where ϖ y x are the Gauss-Hermite weights and nodes for m_z.

Finally we just have to solve the system given by

$$\mathcal{R}(A_i; a) = \hat{c}(A; a) - [(A + 1 - \mu) - \hat{c}(A; a)] \left[\delta \sum_{i=1}^{m_z} \varpi_i h(\sqrt{2}x; a)\pi^{-1}\right]^{-\frac{1}{\gamma}} = 0 \;\; i = 1, \ldots, n_A$$

This is a non-linear system in a. To find a any method can be used to solve non-linear equations. In this case, we use the Galerkin method that implies to solve

$$P_i(a) = \sum_{l_A=1}^{n_A} \mathcal{R}(A_{l_A}, ; a)\psi_i(A_{l_A}) = 0 \;\; i = 1, \ldots, n_A$$

C An Accurate Measure

Once the approximation to the solution is calculated we can measure its accuracy. The most direct procedure is to measure how much it deviates from the zero function of the left hand side of (13). In economic terms a zero deviation represents the difference between consumption over capital when the random variables are A and p and the consumption over capital that should be done by an optimizer agent that knows that tomorrow he will use the rule given by c knowing that the growth of the aggregated personal wealth will be

$(A+1-\mu) - c(A,p)$. Thus, the residual function applied to the approximated solution is the optimization error of a period in terms of consumption over capital. The function

$$E(A,p;a) = \frac{\mathcal{R}(A,p;a)}{\hat{c}(A,p;a)}$$

supplies a free amount of dimension an represents the optimization error as a fraction of the consumption over capital of the period. This function is used as an accuracy index, because this is the way errors are expressed in economic terms. Judd (1998) interprets this relative error of optimization as the irrationality in which agents incur when they use the approximation rule instead of the real rule of optimum decision. If this optimization error were found to be 0.1 it would mean that the approximation implies that the agents make 10 per cent of errors in the decisions of consumption over capital period by period.

The quality of the solution was tested by means of $E(A,p;a)$ in a great amount of (A,p) points that were not used to obtain the solution. Particularly, $\log_{10}||E||_\infty$ that has a clear interpretation is calculated; $\log_{10}||E||_\infty$ is the maximum possible error that can happen. The rule was calculated by using a grid of 6400 points in $[A_m, A_M] \times [p_m, p_M]$, 80 in $[A_m, A_M]$ and 80 in $[p_m, p_M]$.

Model	$\log_{10}(\|\|E\|\|_\infty)$
Reference	-6.1152
Two goods	-4.0606

Table 22: Accuracy measure.

Table 22 shows the values of accuracy measure obtained with the approximations to the solution reference models that is discussed in section 5.8 and for the model of section 3. The accuracy is higher for the reference model than for the model proposed, possibly because the dimension of the space of variables is higher in the latter model. Anyhow, the accuracy of the approximation of the solution is very high in both cases.

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